Does Stock Market Performance Reflect Instability, Risk, or Uncertainty?

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Abstract

This article analyzes the volatility and performance of the U.S. stock market through the lens of debates on informational efficiency. Based on Monte Carlo simulations and regression models applied to the S&P 500 index, it is shown that annual performance and volatility can be largely explained by fundamental variables known ex-ante. The results underscore the practical difficulty of distinguishing between randomness and uncertainty, as both factors jointly influence the required rate of return. The study highlights the existence of an implicit valuation convention, which renders short-term performance partially predictable—a feature that contrasts with the much longer horizons associated with Shiller's cyclically adjusted price/earnings ratio (CA-PE). Furthermore, the article also proposes a new and non-conventional explanation of the ex-ante required return, grounded in the capital accumulation rate and in volatility and inflation, largely derived from observations preceding the investment. These findings suggest that Fama's semi-strong efficiency hypothesis may be called into question, as partial predictability of performance opens up new perspectives for investors able to interpret prevailing market conventions.

JEL : B5, B41, C15, C53, D53, D81, D84, G02, G11, G12, G13, G14, G17

Declaration of Interest

The author declares no conflict of interest. Although the Fairness Finance model cited in this paper was originally developed by the author, he no longer holds any professional or financial ties with the company that owns it.

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Introduction

While the two world wars of the 20th century did not bring down the real economy, some would argue that the only truly mortal threat it faces is financial instability, especially when it takes the form of "irrational exuberance"—to use the expression coined by Alan Greenspan in 1996, after Robert Shiller presented him with a series of indicators pointing to what would later be called the "Internet bubble." It would be a form of blindness to deny the existence of such bubbles. On their psychological foundations and development, theorists from very different backgrounds have, it seems, reached convergent views, such as Robert Shiller (preface to the third edition, 2015) and Hyman Minsky (Ch. 1, 2008). Yet, before the point of no return is reached and the worst occurs, the daily reality of financial markets is to confront volatility. Much like bubbles, volatility itself poses a challenge to the theory of rational expectations, which envisions the formation of expectations about the future as if reading an open book, in the spirit of Muth (1961).

In what follows, we will seek to demonstrate that the analysis of volatility not only challenges this extreme point of view, but also highlights the overly idealized nature of the efficient market hypothesis as formulated by Eugene Fama (1970). The scope of our analysis will be limited to the market for publicly traded equities in the United States, given the ease of accessing relevant data. Due to the format of this work, we will restrict ourselves to the analysis of aggregated information intended to provide an overview of the market as a whole, that is, at the level of the S&P 500 index.

First, we will recall the role that volatility occupies in financial valuation models—models that are often, and mistakenly, presented as being purely of neoclassical inspiration. In reality, these models are the site of an ongoing conflict over how to conceptualize risk and uncertainty. On this occasion, we will see that risk compensation is not necessarily found where one might expect, particularly from the perspective of Post-Keynesian theorists. Second, we will seek to extend Robert Shiller's work on excess market volatility relative to fundamentals by introducing a new approach: modeling stock market volatility with the aim of distinguishing the respective roles of risk and uncertainty, as defined by Knight (1921). Finally, we will analyze stock market dynamics in an effort to identify evidence of an evaluation convention that challenges the random walk hypothesis, which is often considered the purest manifestation of the efficient market theory.

1. Is volatility—supposedly the only trustworthy form of risk—really what the market rewards?

Although volatility lies at the core of two dominant models that shape today's valuation convention, we will see that financial markets do not conform to a simplistic dichotomy between predictable randomness and fundamental uncertainty. In a way, neoclassical and Keynesian approaches jointly preside over the market value of risky financial assets, while the theory of rational expectations continues to exert a disproportionate influence on the teaching of finance.

Due to space constraints, we will not define volatility here. However, the reader may refer to a brief definition provided in Appendix 1.

For now, we note that volatility plays a central role in the "risk-neutral" option pricing model, which might seem to confirm the rational expectations hypothesis. Yet this would be misleading, since the model still depends on an assessment of the underlying asset. As we will show later, the value of a stock relies on a market convention that combines **risk** (randomness) with the principle of fundamental uncertainty to determine the required rate of return based on perceived risk.

As a reminder, given the space constraints, we will simply note here that volatility is also used by regulated financial institutions to determine their capital requirements, through the concept of Value at Risk (VaR). Volatility thus serves both as a symptomatic indicator of market instability, and, through a typically recursive and self-referential process, is simultaneously incorporated by the market itself in the formulation of its own valuation convention.

1.1. The Risk-Neutral Approach: A Project of Risk Domestication?

Anyone who has ever looked into option valuation cannot fail to notice that the entire theoretical framework underpinning it—originating from the seminal work of Black and Scholes (1973)—rests on a powerful illusion: the supposed ability to distinguish clearly between risk and uncertainty.

- In this framework, risk is reduced to randomness, captured by statistical laws whose parameters are assumed to be either stable or evolving along a predictable path. Risk thus appears as the tameable face of uncertainty, made tractable through stochastic calculus.
- Uncertainty, by contrast, is not measurable and is therefore never seriously considered within this theoretical framework. It holds no real interest for it.

In his *Treatise on Probability* (1921) as well as in the *General Theory* (1936), Keynes clearly summarized the impossibility of drawing such a distinction, while restoring the true proportion of any conjecture about the future—that is, any forecasting exercise. This in no way implies that such an exercise, which necessarily precedes action, should be abandoned. Rather, it must be approached for what it truly is, and its implications should be acknowledged—particularly regarding what is fair and reasonable to expect in return from savers, whose preference for liquidity cannot otherwise be properly understood.

Yet in contemporary option theory, uncertainty is so thoroughly excluded that the pricing model is referred to as "risk-neutral", indicating an underlying assumption of risk indifference on the part of the investor. Without delving into the arbitrage[1] and hedging arguments underlying this surprising result, we should retain the following key points:

- The model is based solely on probabilistic paths of the underlying asset's market value, calculated from the volatility of its price return. This requires no analysis of the asset's intrinsic qualities or the dynamics that characterize it. That laborious task is left to the market, which determines the price of the underlying in advance. In this sense, the option is merely a derivative, not a risky financial asset in its own right.
- The Merton model (1974) claims to extend the risk-neutral approach to the valuation of an entire firm for its shareholders, by incorporating its credit risk. In doing so, the boundary between the derivative and the underlying becomes blurred: is not every underlying asset, in a sense, an unrecognized derivative? There is something of the martingale in this conception of risk. By analyzing stock prices much like haruspicy was practiced in ancient Rome, one might claim to predict whether a company will be able to pay its employees or settle its accounts with suppliers in the coming year.
- Since the model—or perhaps even the investor—is risk-neutral, there is no need to require compensation for committed capital in the form of a risk premium. This is thought-provoking, considering that we are dealing with options—arguably among the riskiest financial assets—closely aligned with startup equity, to which they bear a striking resemblance not

[1] To simplify the understanding of this hedging mechanism, let us take the case of stock options. According to Black & Scholes, one must be able to construct a specific portfolio composed of the underlying asset such that the changes in value of the option and of the underlying perfectly offset each other. In this configuration, risk disappears. Since it is impossible to predict the stock price trajectory with certainty, the solution adopted is purely mathematical: it assumes continuous-time rebalancing of the portfolio at infinitesimal intervals. This is a theoretical abstraction that leads to major difficulties when one attempts to approximate it in practice, not least because of the transaction costs involved. To get around this, it is further assumed that, at any given time, market makers who ensure the liquidity of these options have written as many call options as put options. Even so, once we abandon the fiction of infinitesimal rebalancing, we are faced again with radical uncertainty about the future: in fact, it can be shown that such hedging strategies fail in the event of the issuing company's bankruptcy. The market makers who had guaranteed the value of Enron put options know something about that…

only in terms of upside potential and likelihood of failure, but also in their implicit convexity.

- The entire valuation is based on a somewhat overused probability law—namely, the Laplace-Gauss distribution—from which a truncated expectation is derived[2], resulting in a value that appears to rest essentially on an estimate of the asset's volatility.

Thus, in what is considered one of the most sophisticated models of contemporary finance, volatility appears to be the sole risk indicator—both necessary and sufficient to determine the value of an option, and even of a stock, if one follows Merton's reasoning. Few users of this model are aware that it was originally developed by Paul Samuelson (1965), who intended to incorporate a risk premium to compensate the option holder, whose downside risk is significantly higher than that of the stockholder. The solution proposed by Black and Scholes is therefore only a particular case, constructed for academic purposes, and they themselves were reportedly surprised by its widespread success. This success is largely due to the fact that neither Samuelson nor anyone else since has managed to endogenize the formulation of this risk premium. The popularity of the risk-neutral model owes more to a pragmatic anchoring in the absence of alternatives than to the triumph of the rational expectations theory, as is sometimes claimed.

Even if the model were not a poor approximation—except at short time horizons—it still cannot dispense with the market value of the underlying asset, not even in Merton's framework. As we will see, the determination of that value is the battleground between the rational expectations school and those who hold faith in the principle of uncertainty.

1.2. The CAPM: A Synthesis Threatened by the Rational Expectations School?

The dominant model used for corporate valuation as a "market convention"—in the sense given to this term by André Orléan (1999)—is not Merton's risk-neutral framework. As regularly confirmed by surveys—for instance, Bancel et al. (2014)—valuation practitioners and professional investors

[2] A basic call option, for example on a stock, is a promise of capital gain if, at a predetermined future date (t), the stock price S is higher than a strike price X. This gain, "S − X", is the intrinsic value of the option at maturity (the option is exercised only if this value is positive).

Knowing the parameters of the normal distribution assumed to govern the returns of the stock, one can derive the expected future values of the stock conditional on exceeding the threshold X. This is a truncated expectation, $E(S|S > X)$. One can also derive the probability that S exceeds X. These two results allow us to compute the future value V_t of the option: $V_t = p(S > X) \times E(S|S > X)$.

Since the model is risk-neutral, the present value of the option V_0 is obtained by discounting the expected future value at the prevailing risk-free rate r_f at time t = 0: $V_0 = V_t \times e^{-t \times r_f}$.

This logic is quite simple compared to the usual presentation of option pricing models, which are typically burdened with the mathematical complexity inherited from Black & Scholes' original derivation. Their reasoning requires a number of highly imaginative assumptions to justify the use of the risk-free rate—arguably the keystone of their framework. This rate also serves to simulate V_t in the future, as the drift term (see Appendix 1, Equation 12).

remain strongly attached to the discounted cash flow DCF[3]. This model is often referred to as "intrinsic," in reference to John Burr Williams (1938), who advocated it and is traditionally contrasted with the multiples-based approach, often labeled "analogical." It could be shown that any valuation multiple can be derived from a DCF, but that lies beyond the scope of the present discussion. We will therefore focus on the cost of equity capital expected by shareholders, which the market convention continues to estimate primarily using the Capital Asset Pricing Model (CAPM)—a model developed independently in the 1960s by J. Treynor, W. Sharpe, J. Lintner, and J. Mossin.

The CAPM is sometimes seen as a natural extension of neoclassical theory aimed at determining the minimum required rate of return on investments (see Laurent Cordonnier et al., 2018). In reality, it is more than that: it represents a synthesis between the Keynesian principle of uncertainty and the recognition of measurable risk. As such, the CAPM is not merely a dichotomy inherited from Frank Knight's (1921) distinction—it is a genuine fusion of the two dimensions.

It should be recalled that according to the CAPM, the opportunity cost k_s required by the shareholders of a company S is expressed as follows:

$$k_S = \Pi \times \frac{\sigma_S}{\sigma_M} \times \rho_{S,M} + r_f \quad 1)$$

Where Π denotes the market's average weighted risk premium, r_f the risk-free rate corresponding to the investment horizon, $\frac{\sigma_S}{\sigma_M}$ the ratio of the stock's volatility to that of the market portfolio, and $\rho_{S,M}$ the correlation coefficient between the returns of the stock and those of the market portfolio.

To establish an order of magnitude, and using the U.S. listed equity market as a reference, we will assume that over a ten-year investment horizon, the risk-free rate r_f has fluctuated between 2% and 3% over the past decade—let us say 2.5%—and that the market's average internal rate of return has remained relatively stable around 6%[4]. Consequently, the equity risk premium, defined as the difference between the latter and the risk-free rate, lies between 3% and 4%. This result stems from

[3] DCF stands for "Discounted Cash Flow." The term *cash flow* is a conventional expression used among finance professionals. In the absence of an explicit specification, it refers—depending on the context—either to the cash flow available to equity holders, or to the cash flow to the firm, i.e. available to all capital providers, i.e., both shareholders and financial creditors such as banks and corporate bondholders.

[4] This is an estimate of the net IRR based on analysts' forecasts, adjusted for their optimism bias. For further details, please refer to the Fairness Finance website.

the Fairness Finance[5], market model and is supported by other sources[6], including some public ones, such as Aswath Damodaran (2024)[7].

Since the cost of equity is not declarative, it can only be estimated. Calculating an internal rate of return (IRR) based on an intrinsic valuation model is considered here a satisfactory means of estimation, as suggested by our previous work (Clère – 2025). According to this model, the expected cost of equity for a stock is higher than the risk-free rate[8], the latter being, in Keynes's view, the price of liquidity forgone—a price that increases with the investment horizon and the weight of fundamental uncertainty.

The relative volatility $\frac{\sigma_S}{\sigma_M}$ of listed stocks is generally higher than that of the market portfolio. Conversely, the correlation coefficient $\rho_{S,M}$ lies between zero and one. It can be shown that, on average—weighted by the market capitalizations of the constituent securities—the product of the relative volatility and the correlation coefficient (commonly referred to as the beta coefficient[9]) equals one. In other words, volatility is not the primary explanatory variable for the cost of equity, nor for the value of a stock. Empirically, beta coefficients for individual stocks are generally found to lie between 0.5 and 1.5. In the case of the U.S. market, where the expected equity risk premium can reasonably be estimated at around 3.5%, the effect of volatility, as modulated by correlation, thus ranges between ±1.75%—that is, approximately half the market premium. This implies that the required individual IRRs will typically range from 4.25% for companies with a beta of 0.5 (2.5% risk-free rate plus 1.75% individual premium) to 7.75% for companies with a beta of 1.5 (2.5% + 5.25%), with most observations clustered around 6%[10].

Moreover, the value of a stock also depends on financial expectations regarding the company. We will return to this point later when attempting to simulate volatility. At this stage, it can be assumed

[5] Model developed by the author in collaboration with Stéphane Marande, under the supervision of Pierre Béal and with the financial support of the BM&A firm.

[6] The estimate can be derived either from (i) the observed ex-post difference between stock returns and the risk-free rate, or (ii) the anticipated difference between the market's expected IRR and the risk-free rate. In the Fairness Finance model referenced here, the IRR is calculated based on forecasts made by financial analysts regarding the cash flows of publicly listed companies. Other commercial models produce ex-ante IRR estimates based on similar principles, such as the Trival model by Associés en Finance, the Valphi model, or the CCEF model.

[7] The estimated IRR as of January 2024 by A. Damodaran stands at 8.5% (see p. 101), a level comparable to the gross IRR estimated by the Fairness Finance model at 7.9%.

[8] Indeed, all parameters are positive, including the correlation coefficient; negative observations are rare and incidental. The risk-free rate here refers to the yield on 10-year U.S. Treasury Bonds.

[9] The beta coefficient can also be expressed as the ratio of the covariance of returns to the variance of market returns: $\beta = \frac{\sigma_S}{\sigma_M} \times \rho_{S,M} = \frac{Cov(S,M)}{Var(M)}$.

[10] This analysis considers only large-cap companies, whose expected IRR—adjusted for optimism bias—is unlikely to be affected by a liquidity premium. Typically, this refers to U.S. companies with a market capitalization exceeding USD 7 billion.

that, in practice, forecast revisions and the degree of confidence in those forecasts likely contribute to the individual volatility of stocks—volatility that, in turn, may influence their cost of equity in a recursive process reminiscent of the adaptive expectations theory.

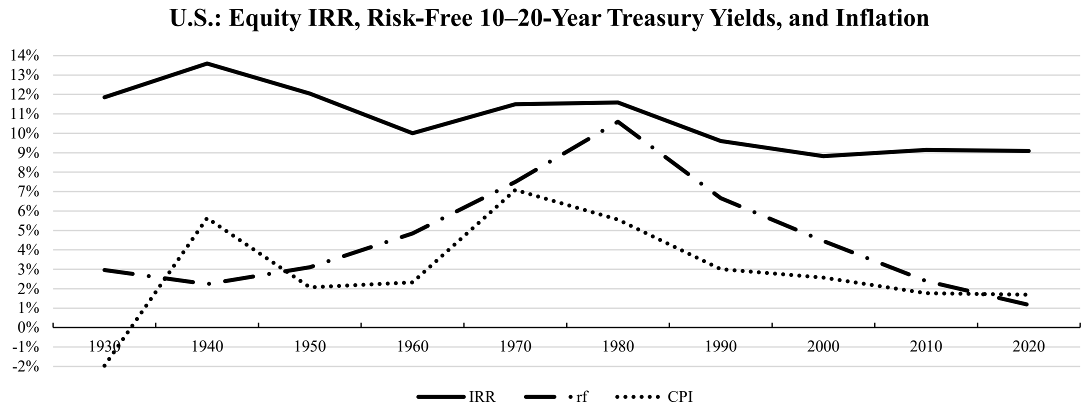
In any case, what distinguishes the CAPM from the risk-neutral model is not so much the more modest role played by volatility, but rather the explicit inclusion of a risk premium. Estimated, as noted earlier, at between 3% and 4%, this premium is a model input that is not endogenized. The CAPM does not specify the nature of the market risk premium; it merely observes it. Its fluctuations can either amplify or dampen, through the beta mechanism, the effect of volatility on individual premiums. Meanwhile, volatility affects only part of each premium: there appears to be an irreducible individual component, accounting for roughly 50% of the average equity risk premium in the U.S. market.

It is well known that this premium poses a problem for proponents of the rational expectations theory. Indeed, if risk were reducible to pure randomness, it could be managed either through hedging instruments or by relying on the central limit theorem (CLT) to ensure that investments converge toward their expected returns. Conversely, if risk stems from radical uncertainty, the rational expectations hypothesis becomes untenable. The attack on the equity risk premium by the rational expectations school was therefore inevitable, and it was structured around the famous article by Mehra and Prescott (1985), which argued that in the theoretical framework of the consumption-based CAPM, the premium should not exceed 25 basis points (0.25%)—less than one-tenth of its observed value. This was followed by a research program aiming to empirically confirm its forthcoming disappearance (see Jagannathan, 2000). These studies emerged shortly after Paul Volcker's tenure as Chairman of the Federal Reserve, during which a sharp rise in interest rates was imposed, with every likelihood of temporarily compressing the risk premium. As shown by Roland Clère (2025), since that episode, not only has the risk premium[11] not declined, but it likely continued to rise, as illustrated in Figure 1 below, where it is represented by the gap between the market's gross IRR[12] (k) and the risk-free rate (rf):

[11] The risk premium is derived from a panel of annual time series estimating the IRR of the U.S. equity market, based primarily on four academic sources: Jagannathan et al. (2000), Ke et al. (2014), Mishra et al. (2018), and Fairness Finance. The premium is calculated by subtracting the return on 10-year Treasury Bonds from the estimated IRR.

[12] The gross IRR differs from the net IRR by including a risk premium to correct for the optimism bias in cash flow forecasts used to estimate the IRR. According to the Fairness Finance model, this premium has been around 2% over the past ten years.

Figure 1



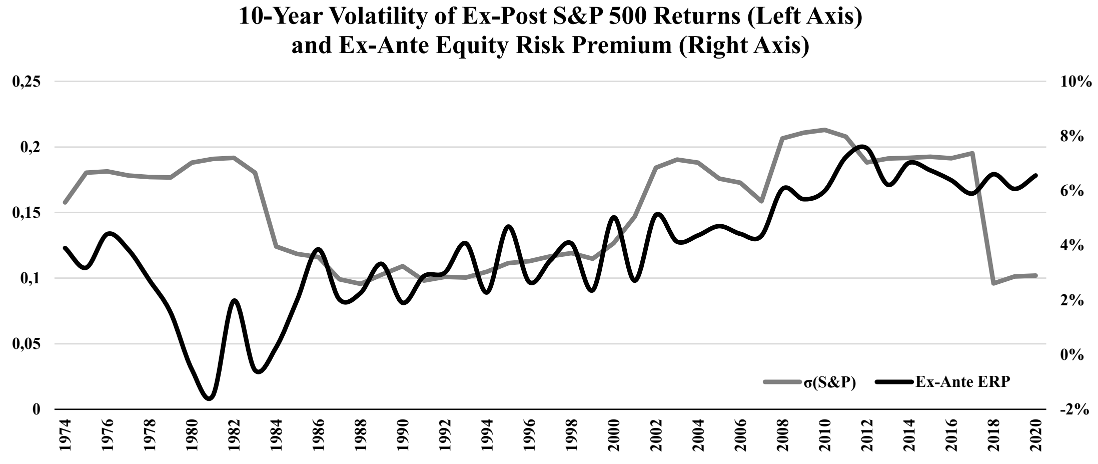
Where *k*, *rf*, and *CPI* refer respectively to the ten-year averages of the U.S. market's ex-ante IRR, the risk-free rate, and the change in the Consumer Price Index. Yields on 20-year Treasury bonds are used for *rf* prior to 1970, and 10-year Treasury yields thereafter.

For the rational expectations school, since the premium cannot be based on volatility, it must arise from the principle of uncertainty. But how can this hypothesis be explored?

1.3. The Role of Uncertainty in Risk Compensation

While the equity risk premium has seemingly increased in the United States since 1981, no comparable rise is observed in classical risk, i.e., the volatility of market portfolio returns. For the S&P 500, this volatility—calculated over rolling 10-year periods—has fluctuated within a range of 10% to 20% since the post-war era. As suggested by Figure 2 below, compiled by Roland Clère (2025), the evolution of the ex-ante risk premium does not follow that of the ex-post return volatility of the index. In fact, when S&P 500 volatility reached its peak between the two oil shocks, the ex-ante risk premium was at its lowest recorded level in the 20th century. At best, one might consider that volatility contributes—alongside other variables—to the level of the risk premium, a hypothesis that remains to be demonstrated and which we address later in this article, particularly in Section III.

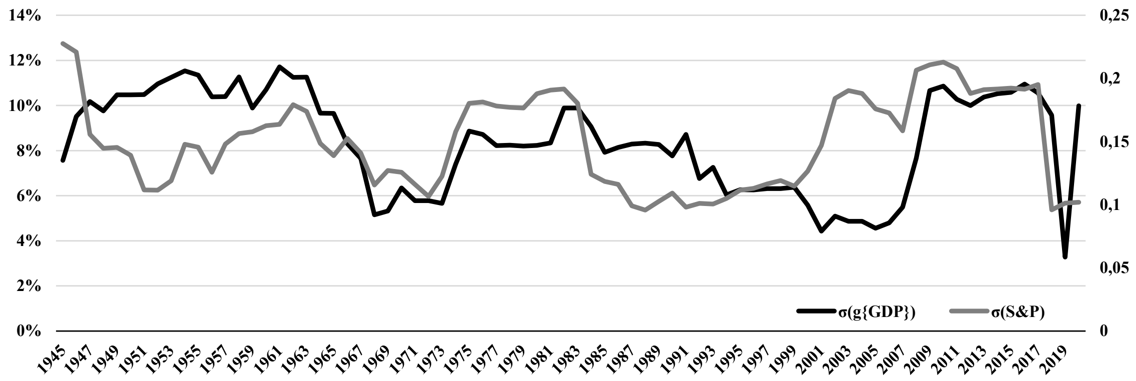
Figure 2



As shown in Figure 3 (see Clère – 2025), since the post-war period, stock market volatility—when calculated over rolling 10-year windows—tends to mirror the (rescaled)[13] coefficient of variation of U.S. per capita GDP growth, while also displaying a few distinct episodes, such as the Internet bubble. Under these conditions, the ex-ante equity risk premium cannot be seen as a mere reflection of ambient economic instability—unlike market volatility—and should instead be understood as stemming from the anticipation of future risk. But is not the defining feature of the future precisely its uncertainty, especially when it comes to stock market performance?

Figure 3

10-Year Ex-Post Coefficient of Variation of U.S. Real GDP per Capita Growth (Left Axis) and S&P 500 Return Volatility (Right Axis)



2. Does Volatility Contradict the Theory of Rational Expectations: Is It Excessively High?

2.1. A Brief Review of the Literature on the Endogenous Origins of Volatility

As previously discussed, volatility appears to reflect the instability of the economy, yet it does not explain the long-term risk premium demanded by investors, which seems instead to stem from the principle of fundamental uncertainty. One may wonder whether this volatility is purely informational in origin. Several authors, in fact, suspect that part of volatility arises from the functioning of the market itself, which by its very nature amplifies the impact of information flows on price movements. Volatility would thus potentially have a partly endogenous (or functional) origin, resulting from the market's operation and the interactions among its participants.

Fama (1965), French (1980), and French and Roll (1986) compared the volatility of listed securities during the week with that observed between the close on Friday evening and the opening on

[13] The coefficient of variation is defined as the standard deviation of 10 annual logarithmic changes in GDP per capita, divided by the mean of these 10 changes. For the sake of comparison with the 10-year volatility of the S&P 500 index—and in a gesture to statistical convenience—this latter figure is divided by 10.

Monday morning. They found that weekend volatility was much lower than expected compared to weekday volatility. Roll (1984) confirmed this result for orange juice futures contracts, whose prices, according to him, are essentially explained by weather conditions.

Ramos, Bassi, and Lang (2020–23), using an agent-based and stock-flow consistent model, reconstruct a foreign exchange market in which investors are divided between fundamental analysts and chartists—the latter lacking a decision-making model based on economic data and relying solely on actual exchange rate movements. Through this framework, the authors are able to generate volatility clusters, bubbles, and crashes, and, more generally, a level of volatility that exceeds what would be expected from fundamental economic information.

Robert Shiller (1981) offers further insight by using volatility to test the central hypothesis of the rational expectations theory, namely that forecasts should equal realizations. For this purpose, he employs a dividend discount model. After a certain number of terms, the residual value becomes negligible, and the present value of a stock is based on a weighted average of discounted dividends. Without delving further into the details of these calculations, such a demonstration essentially amounts to comparing the variance of an average to that of the averaged variable—a result that is, in a sense, a foregone conclusion. Nevertheless, using the central limit theorem, it confirms the common intuition that if the future were known, there would undoubtedly be less volatility…

We choose here to revisit this question by taking a somewhat different approach, making use of a tool that was not available to Shiller: an estimate of the internal rate of return (IRR) required by the market.

2.2. Decomposing Price Growth to Identify the Source of Volatility

Since the expected internal rate of return (IRR) is published by various research firms and by all major investment banks, it can be regarded as a widely shared market indicator among professionals—on the same level as the average price-to-book ratio (P/B) of the market portfolio and the growth forecasts issued by major institutions such as the IMF. With this readily available information, it is thus possible to calculate a theoretical value for the index representing the market portfolio using the following formula:

$$P_t = B_t + B_t \times \frac{\overline{ROE}_t - TRI_t}{TRI_t - g_t} \quad 2)$$

Where B_t denotes, at date t, l the book value, i.e. the amount of shareholders' equity, $\overline{ROE}_t$, the normative accounting return on equity, i.e. the normative earnings relative to equity[14], the cost of equity capital required by investors—assumed to correspond to the estimate of the net ex-ante IRR—and finally, the expected growth rate of equity, g_t.

This formula is supported by Pablo Fernandez (2003), whose work establishes the equivalence between the Gordon and Shapiro model (1956)—which involves discounting a firm's normative dividend in perpetuity—and Equation (2), which consists in capitalizing the rent it generates on its equity and adding it to its book equity. The rent here is defined as the difference between the firm's normative return on equity (ROE) and its cost of equity capital, or required IRR. It determines the existence of a price-to-book ratio (P/B) greater than one. Indeed, from Equation (2), one derives the expression for the equity multiple, P/B, or "price to book":

$$\frac{P_t}{B_t} = 1 + \frac{\overline{ROE}_t - TRI_t}{TRI_t - g_t} \qquad 3)$$

Given that, by definition, $P = B \times P/B$, he logarithmic return of a stock resulting from a price change between date *t* and date $t + 1$ can thus be written as:

$$ln\frac{P_t}{P_{t-1}} = ln\frac{B_t}{B_{t-1}} + ln\frac{P_t/B_t}{P_{t-1}/B_{t-1}} \qquad 4)$$

$$\Leftrightarrow \quad \frac{\dot{P}}{P} = \frac{\dot{B}}{B} + \frac{\dot{P/B}}{P/B} \quad \Leftrightarrow \quad g\{P\} = g\{B\} + g\{P/B\} \qquad 5)$$

It thus appears that the growth in price $g\{P\}$ can be decomposed into the sum of the growth in book value, denoted $g\{B\}$, and the appreciation of the valuation multiple $g\{P/B\}$.[15]

The growth in equity capital should be seen as reflecting the growth of productive capacity and business activity, assuming stable capital intensity and leverage ratios—an assumption that is arguably reasonable over a 12-month horizon and at the aggregate level of the stock market as a whole. Over the long term, however, these ratios may shift. In the presence of cointegration, the growth in corporate value should mirror that of the overall economy. Indeed, any advocate of the rational expectations hypothesis would concede that the expected appreciation of the valuation

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[14] ROE stands for return on equity. It is defined as: $ROE_t = E_t/B_{t-1}$ where t denotes the fiscal year, E (earnings) represents the net income, and B (book value) denotes the shareholders' equity as of the previous fiscal year-end.

[15] To denote annual variations, one could have used $\dot{P}/P$ instead of $g\{P\}$, $\dot{B}/B$ instead of $g\{B\}$, and $\dot{PB}/PB$ instead of $g\{P/B\}$. However, we choose the notation with "g" to more clearly refer to the mean and standard deviation. For example, from the random variable $g\{P\}$, one obtains $\bar{g}\{P\}$ et $\sigma\{g\{P\}\}$.

multiple is zero—unless one anticipates an acceleration in growth or a change in the cost of capital, which would be a most refined way of predicting the future…

It is worth noting in this regard that the U.S. stock market has exhibited, since 1944, a real annual growth rate of 3.7%[16], compared to just 2.7% for the economy as a whole. This discrepancy raises a number of questions that we can merely signal here for lack of space, but which we attempt to address—at least in part—by examining the long-term evolution of the cost of capital in our previously cited article (see Clère – 2025).

In any case, at this stage of the analysis, it is possible to perform a Monte Carlo simulation of the volatility of the market's logarithmic returns based on three variables: the market IRR, the average ROE of the constituent firms, and the estimated growth rate.

2.3. Simulating Price Volatility from Its Explanatory Variables

A detailed presentation of the model can be found in Appendix 2. It should be noted that the volatility simulated here pertains to the S&P 500 index, which is intended to reflect the U.S. stock market portfolio weighted by market capitalization. To this end, we proceed as if annual estimates of the three key parameters were available from the end of 2014 to the end of 2022, thereby constructing a training dataset. Indeed, it is from 2014 onwards that an estimate of the market IRR has been produced by the Fairness Finance model. It should also be noted that the average ROE of the index's constituent firms is calculated here based on data predating 2014.Given their respective distributions and the empirical correlations observed between them, we first generate two sets of simulations of the index's 12-month variation. Each set differs in the degree of optimism embedded in the 12-month equity growth assumption, $g\{B\}$:

- The optimistic scenario adopts as its central tendency the average growth rate of equity capital, $\bar{g}\{B\}$, observed between 2014 and 2019, namely 5%. This empirical average of annual growth rates is calculated by excluding the 2020 health crisis and the aftershocks of the subprime crisis. It is not statistically different from the average computed over the 2004–2022 period, which stands at 4.8% and encompasses these phases of instability. Furthermore, the standard deviation of this growth rate—its volatility—is estimated at 7.3%, corresponding to the level observed over the 2004–2022 period. In other words, we attempt to construct a heuristic in the sense of Tversky and Kahneman (1974), of the sort a typical market participant might intuitively employ. We proceed here *as if* the central tendency

[16] Robert Shiller's official website, and for U.S. GDP: J.P. Smits, P.J. Woltjer, and D. Ma (2009), *A Dataset on Comparative Historical National Accounts, ca. 1870–1950: A Time-Series Perspective*, Groningen Growth and Development Centre Research Memorandum GD-107, Groningen: University of Groningen. Website.

could be estimated "more reliably" by excluding extreme fluctuations that distort its precision: the "true" growth rate would thus be best captured during "normal" market phases, simply because they are more frequent, whereas periods of severe instability are too rare, statistically speaking, to avoid distorting the mean. However, there is no reason to assume that the market will be any less volatile in the future. This bias is meant as a cautious response to uncertainty, aligned with Minsky's (2008) warning against the illusion of stability. Such reasoning is assumed to reflect the typical posture of *animal spirits*, in Keynes's (1936) sense—torn between optimism and fear of uncertainty. We shall therefore describe this mindset, and the resulting scenario, as both "optimistic and prudent," respectively in terms of return and risk.

- The absence of anchoring and the lack of certainty are inherent to any forecasting exercise for those who cannot meaningfully influence the future. Faced with the unknown, *animal spirits* thus rely on experience. Since living organisms spend their lives adapting, it is unclear why the principle of adaptive expectations should be rejected outright in this context. We will therefore assume that forecasting is a form of arbitration between the hypothesis of a return to the central trend[17] and the recognition of the emergence of a new growth regime. What is described here is, of course, the zero degree of anticipation—when forecasting becomes social consensus. The true added value—some would call it divination—lies in the ability to detect, or even anticipate, regime shifts before the crowd does. The topic is an old one, already addressed by Homer through the figures of Cassandra and Laocoön. Closer to our time, it is regrettable that Hyman Minsky passed away just before one of the most emblematic episodes of "irrational exuberance" in financial markets, and thus could not put his understanding of collective market psychology into practice to warn his contemporaries about what was taking shape. Theorists, however, do not necessarily make the best forecasters: Keynes himself failed to anticipate the October 1929 crash, despite devoting much of his time to deciphering the expectations of his contemporaries—and being a remarkably skilled speculator in his own right. The privilege of having played the role of a modern-day Cassandra thus belongs to Robert Shiller[18], the emblematic thinker of the behavioral school. Although he identified both the Internet bubble and the subprime bubble before they burst, he was, of course, no more heeded than Cassandra or Laocoön. He was nonetheless rewarded for his commendable efforts in 2013 with a prize awarded by

[17] The temporary nature of deviations from the central trend is the convenient explanation offered for any forecasting error. When a phenomenon is stationary, error becomes forgivable.

[18] By comparison, Nouriel Roubini can claim "only" the prediction of the subprime crisis as a highlight in his track record.

the Swedish central bank in memory of Alfred Nobel—shared, ironically, with Eugene Fama, who has consistently refrained from attempting to predict market movements, in line with the efficient market theory he helped develop.

It is within this framework that we now introduce a second scenario available to the *animal spirit*—this time based solely on recent experience, namely the 2014–2022 period. During this time, a marked slowdown in capital accumulation is observed, with average growth, $\bar{g}\{B\}$ falling to just 3.7% per year. Moreover, we shall proceed *as if* we were unaware of Minsky's "paradox of tranquility." Accordingly, the volatility of the accumulation rate is set equal to the empirical average over the same period, namely 2.9%. This would imply a regime shift in capital accumulation between the early 2000s and the COVID-19 crisis. We shall refer to this scenario as "pessimistic and myopic."

The results of these two scenarios, along with a third intermediate one, are presented in the table below. It also includes the actual data for the S&P 500:

Table 1

	Scenarios			Scenarios	
Theoretical Return and Its Volatility According to the Accumulation Scenario	Optimistic Prudent	Pessimistic Myopic	Pessimistic Prudent	S&P 500 2014-2022	S&P 500 2004-2022
Hypotheses:					
Book Equity Growth g{B}	5,0%	3.7%	3.7%	3.7%	4.8%
Volatility of Book Equity Growth σ{g(B)}	7.3%	2.9%	7.3%	2.9%	7.3%
Results:					
Target Market Return: : $R^L = \ln[E(P_t)/P_0]$	6.2%	4.5%	4.7%	8.1%	6.7%
Target Market Volatility: σ	13.7%	11.1%	13.7%	14.2%	18.8%
Implicit Drift: $\mu = E(\ln P_t/P_0) = R^L - 0{,}5.\sigma^2$	5.3%	3.9%	3.8%	7,0%	4.9%

- The first scenario (from left to right) corresponds to the "optimistic and prudent" mindset. It amounts to betting on a return to the average pace of capital accumulation and, sooner or later, a resurgence of instability, with crises comparable to those of 2009 or 2020. It leads to an expected return on the U.S. market of 6.4% per year—close to the actual return of the S&P 500 from 2004 to 2022 (6.6%). Notably, this rate is equal to that anticipated by the Fairness Finance model. Moreover, this scenario yields a theoretical market portfolio volatility of 13.7%, which is very close to the average observed volatility between 2014 and 2022 (13.4%).
- The next scenario is labeled "pessimistic and myopic." It anticipates a break in the pace of capital accumulation that would persist at a durably lower level than in the early 2000s—a form of economic malaise. Myopia here arises from the belief—likely mistaken—that

instability could remain permanently subdued. Such a scenario would illustrate the human tendency to be deceived by the illusion of tranquility. It leads to an expected market return of 4.6%, with a volatility of 11.1%. These outcomes are noticeably lower than the average market parameters observed in recent years.

- Finally, we present a third scenario, combining low growth with renewed instability. This scenario results in an expected return of 4.8%—only marginally higher than the previous one—paired with a volatility of 13.7%, identical to that of the first scenario.

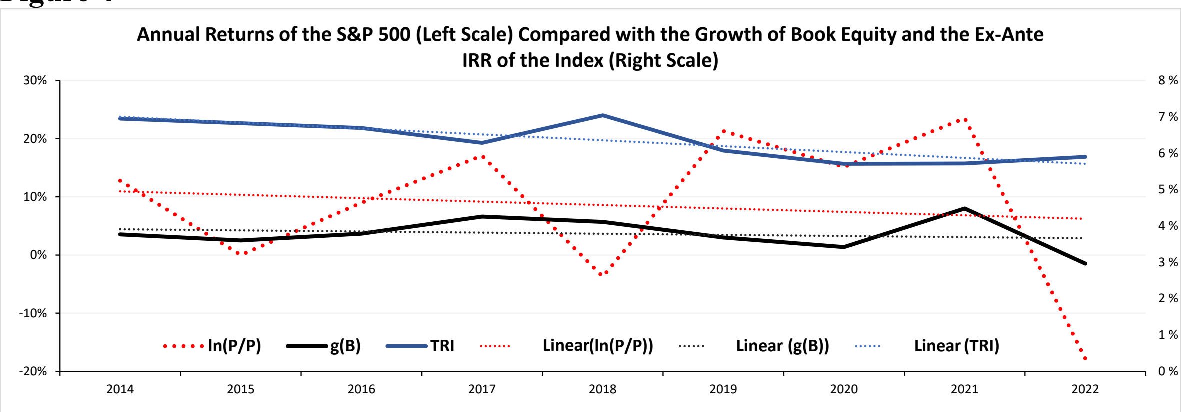
As shown in the previous table, the optimistic (in terms of return) and prudent (in terms of risk) scenario yields results that are closest to those actually observed for the S&P 500 index. However, a certain internal inconsistency becomes apparent. While the expected return aligns reasonably well with the long-term market average (2004–2022), the volatility it implies corresponds more closely to the recent period (2014–2022). If we consider only post-2014 data—which underpins the "pessimistic and myopic" scenario—the modeled return should average 4.5%, compared to an actual observed return of 8.1%. Over the same time frame, expected volatility would be 11.1%, whereas in practice it proved nearly 28% higher, reaching 14.2%.

To interpret these results, let us return to Equation (2), which links price to book value and to goodwill, and consider the evolution of its parameters:

$$P_t = B_t + B_t \times \frac{\overline{ROE}_t - TRI_t}{TRI_t - g_t} \tag{2}$$

As shown in Figure 4 below, between 2014 and 2022, the net required IRR appears to have declined by nearly 100 basis points (1%). All else being equal, this decline would have generated an unanticipated excess return in the stock market. Meanwhile, the target ROE seems to have increased, further amplifying the effect of the lower discount rate and resulting in a sharp inflation of the price-to-book multiple.

Figure 4



Indeed, as shown in Table 2 below, the model's parameters—at least as they can be estimated—appear to have evolved very favorably between 2014 and 2022[19], with the P/B multiple rising from 2.83× to 4.02×. Thus, the estimated annualized (logarithmic) stock market performance of 8.1% can be broken down into 3.7% attributable to equity growth and 4.4% to the expansion of the valuation multiple:

Table 2

S&P 500	31/12/2014	31/12/2022e	2022^
P	2 054	3 912	2 969
B	726	974	974
P/B	2,83	4,02	3,05
E(ROE)	13,7%	14,7%	14,7%
E(IRR)	6,8%	5,9%	6,8%
E(g')	3,10%	2,98%	2,98%

As shown in the fourth column of Table 2, using the ROE and expected growth rates for 2022 but keeping the IRR unchanged from its 2014 level, the P/B multiple would have reached only 3.05× in 2022. Accordingly, the stock market performance would have been reduced from 8.1% to just 4.6%. In other words, the decline in the ex-ante required IRR appears to have had a substantial impact on the ex-post return of the index over the 2014–2022 period. This trend likely also amplified the return volatility observed during that time.

Consequently, had the IRR remained stationary, the return would probably have been closer to those modeled in the "pessimistic" scenarios of Table 1, which correspond to an anticipated capital accumulation rate between 3.5% and 4% per year. In other words, over the period for which IRR estimates are available, the average return should have been around 4.6%—a level nearly identical to the modeled value of 4.5%. While the modeled volatility stands at 11.1%, it is difficult to determine at this stage whether the observed gap of 3.1 percentage points relative to actual volatility is due to the amplifying effect of the declining IRR, or whether it reflects a sui generis excess volatility intrinsic to the functioning of equity markets, as suggested by some of the studies previously cited.

The decline in the net IRR after 2014 can likely be attributed to a reduction in the level of uncertainty perceived by investors. It is conceivable, in fact, that confidence was restored after 2014, as systemic risks stemming from the subprime crisis gradually receded and waves of liquidity were injected by central banks. From this perspective, the 2014–2022 period appears to exhibit an anomaly, highlighted by the decomposition of market performance into the portion attributable to

[19] The 2022 data were estimated as of end-March 2022. For the sake of simplicity, the index value at the end of March was assumed to be equal to its future value at the end of December.

volatility and the portion stemming from a persistent trend—or *drift*, as defined in Appendix 2, section (c).

In theory, this latter component, as estimated in the last row of Table 1, should be relatively close to the capital accumulation rate under the proposed model. In practice, this appears to hold over the long run (2004–2022), as evidenced by the index data (last column of the table). However, for the 2014–2022 period, the central trend parameter would have been 7% per year—nearly double the capital accumulation rate, which stood at just 3.7%. This discrepancy suggests that the main explanatory parameters of stock market dynamics were not stationary during this period, giving rise to a windfall effect.

To better understand this dynamic, it seems appropriate to investigate the relationship between the IRR and capital accumulation. This will allow us to build upon our previous work, which had not yet incorporated the capital accumulation rate as an explanatory variable in stock market dynamics (see Roland Clère – 2025).

3. Revisiting the Valuation Convention and the Predictability of Stock Market Performance

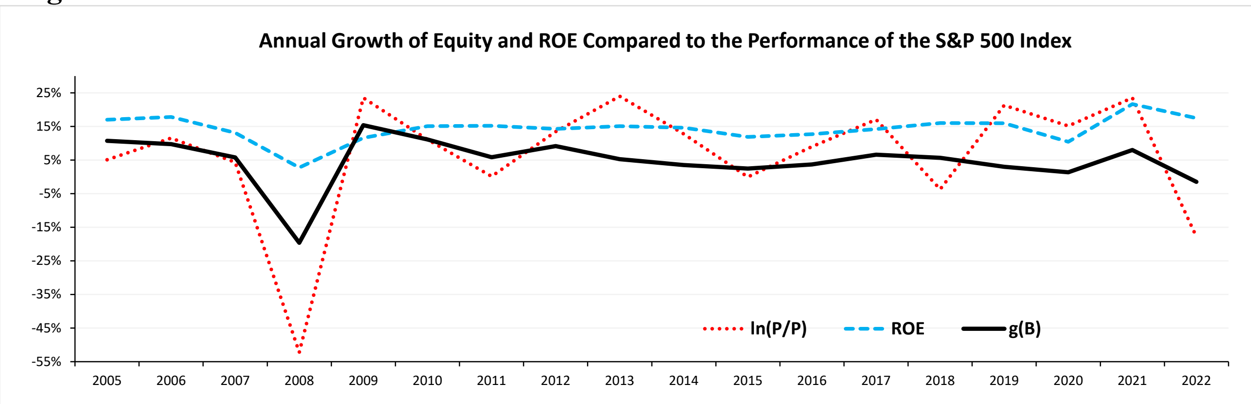
Previously, based on Equation (5)—which theoretically decomposes the variation in a stock's price and is itself derived from Equation (2), which breaks down its value—we simulated the volatility of returns on the U.S. stock market index in order to account for the levels recently observed. At this stage of our analysis, it now seems plausible to consider that the capital accumulation rate, $g\{B\}$, could also serve as an explanatory variable for the annual ex-post returns of the equity market as observed in practice. With this in mind, we will first conduct an empirical test aimed at modeling ex-post performance based on $g\{B\}$ in combination with the other variables previously identified in our earlier explanatory efforts (see Roland Clère – 2025).

In a second step, provided that the hypothesized deductive link is confirmed by the test, we will infer a possible rule for calibrating the ex-ante IRR. This rule would offer an unexpected explanation of the equity risk premium, insofar as it would depart from the standard equilibrium model by relying instead on the capital accumulation rate and the volatility of market returns.

3.1. Simulating Market Performance from Ex-Ante Explanatory Variables

Figure 5 below illustrates the evolution of the annual ROE of the S&P 500 index, the growth rate of equity capital $g\{B\}$ and the index performance, $ln(P_{t+1}/P_t)$, between 2005 and 2022. The examination of these curves appears to support the hypothesis of a behavioral rule influencing stock market dynamics.

Figure 5



To confirm this hypothesis, we estimate an ordinary least squares linear regression in which the dependent variable is the annual logarithmic change in the price of the S&P 500 index, $g\{P\}_{t+1}$, i.e., its ex-post performance measured over a rolling one-year window subsequent to the investment date t. For this purpose, we rely on a dataset computed as of March 31 each year, covering the period from March 31, 2015, to March 31, 2024 (nine observations). This reporting date appears particularly well suited to limiting anticipation bias that would arise from incomplete knowledge of the financial statements for the previous fiscal year, in particular the amount of shareholders' equity at the start of the new fiscal year. Compared to the approach taken by Eugene Fama and Kenneth French (1992) in their analysis of stock market performance, this choice moves the calculation date forward by one quarter, taking advantage of the regulation adopted in September 2002—effective for fiscal years from 2005 onward—which requires annual reports to be published by the end of March[20].

The explanatory variables are calculated over the period preceding the investment date. They are therefore likely to serve as leading indicators of index performance. There are five such variables:

- $\Delta g\{B\}_{t-1}$ — The change in the differential between the growth rate of shareholders' equity and per capita GDP in the year preceding that of the dependent variable. This constitutes a relative indicator of the acceleration in capital accumulation. To emphasize this effect, the annual growth of shareholders' equity is computed as a slightly weighted average μ_t based

[20] Historically, publicly listed companies subject to SEC oversight were required to publish their annual reports within 90 days. This deadline was reduced to 75 days starting with the publication of 2005 annual financial statements, which had to be released before March 15. Beginning with the 2006 financial statements, large companies (with a public float exceeding USD 700 million) were required to comply with a 60-day deadline. Only very small companies (with less than USD 75 million in public float) are still allowed a 90-day reporting period.

on an exponential smoothing rule applied to quarterly data[21]. From this, m_t, the unweighted annual growth of GDP, is subtracted. This growth differential d_t ($d_t = \mu_t - m_t$) is then compared to the previous year's differential d_{t-1}, yielding $\Delta gB_{t-1} = d_t - d_{t-1}$.

- $\Delta g\{P\}_{t-1}$ — The acceleration in the growth rate of the stock market index price. It is likewise measured over the one-year rolling period $t-1$ preceding the investment date, compared to period $t-2$.
- $g\{P\} - ROE_{t-1}$ — The extent to which the growth rate of the index exceeded the return on equity (ROE) during the year preceding the investment. As illustrated in Figure 5, this variable acts as a mean-reversion force on the ex-post variation of the index. In other words, if the market indeed regards ROE as a benchmark for anticipating the pace of capital accumulation and the central tendency of stock market performance, then when performance "gets ahead" of ROE, the market would subsequently tend to underperform. This phenomenon appears to provide evidence of cointegration between stock market growth and the growth of productive capacity, for which shareholders' equity B is a proxy.
- P/E_{t-1} — The price-to-earnings multiple for fiscal year $t-1$ observed at the investment date, as calculated on the companion website to Robert Shiller's work, based on the monthly value of the S&P 500 index. To scale it consistently with the other variables, the multiple is divided by 100.
- IRR_t — The required rate of return at the investment date.

The correlations among these variables provide a preliminary understanding of the general orientation of the model.

Table 3

ρ	$g\{P\}_{t+1}$	$\Delta g\{B\}_{t-1}$	$\Delta g\{P\}_{t-1}$	$[g\{P\}\text{-ROE}]_{t-1}$	P/E_t	IRR_t
$g\{P\}_{t+1}$	100%	38%	2%	-29%	24%	36%
$\Delta g\{B\}_{t-1}$	38%	100%	44%	13%	48%	-12%
$\Delta g\{P\}_{t-1}$	2%	44%	100%	90%	84%	-40%
$[g\{P\}\text{-ROE}]_{t-1}$	-29%	13%	90%	100%	77%	-48%
P/E_{t-1}	24%	48%	84%	77%	100%	-53%
IRR_t	36%	-12%	-40%	-48%	-53%	100%

[21] Where each preceding quarter is assigned a weight equal to 90% of the following quarter, thereby giving relatively greater weight to the most recent quarters.

As shown in the table above, the ex-post variation of the index, $g\{P\}_{t+1}$, is positively correlated with all variables except for the relative lead taken ex-ante by the price change over the ROE, denoted $g\{P\} - ROE_{t-1}$. The presence of such a negative correlation makes it possible for a market dynamic with turning points to emerge. This phenomenon is also accompanied by two unexpected correlations: (i) there is no negative autocorrelation between the ex-ante acceleration of prices, ΔgP_{t-1}, and ex-post market performance; and (ii) the ex-ante P/E ratio does not operate here as a valuation indicator of expensiveness, but rather as a parameter stimulating growth. These results may be specific to the 2015–2024 observation window and to the end-March timing of the annual measurements.

As for the required rate of return (IRR), whose effect is a priori ambiguous, its promise of yield here outweighs its role as a signal of risk aversion. It should be noted, however, that it is negatively correlated with the other ex-ante variables, as expected, including with the lead over ROE, insofar as the latter is closely correlated with ex-ante stock market performance.

The model's parameters and characteristic statistics are summarized in the table below:

Table 4

					AIC =	-56,3
$g\{P\}_{t+1}$	**R^{2*} =**	**94,4%**	F =	28,2	p(F) =	1,0%
	IRR_t	P/E_{t-1}	$[g\{P\}\text{-}ROE]_{t-1}$	$\Delta g\{P\}_{t-1}$	$\Delta g\{B\}_{t-1}$	Cst
Coefficients	**19,86**	**7,87**	**-2,28**	**0,90**	**-2,32**	**-3,13**
Standard errors of coefficients	4,22	1,02	0,30	0,21	0,62	0,42
R^2 and standard error of residuals	**97,9%**	3,91%				
F-statistic and degrees of freedom	28,2	3				
Explained and residual sums of squares	21,53%	0,46%				
One-tailed t*, two-tailed t	t	t	t	t	t	t
Student's t-statistic	4,70	7,68	7,59	4,21	3,73	7,47
p(Coefficient) = 0	1,8%	0,5%	0,5%	2,4%	3,4%	0,5%
VIF (Variance Inflation Factor)	1,5	4,5	12,3	13,9	2,9	

This model appears capable of explaining 97.9% (adjusted R^2 of 94.4%) of the variance in the annual returns of the S&P 500 index over the analyzed period. It is based on a very limited training sample, which raises concerns about the risk of overfitting and coefficient instability due to collinearity among the variables, as indicated by the variance inflation factors (VIF) and Table 3.

Robustness analysis, however, shows that the model's explanatory power remains stable: the adjusted coefficient of determination stays high (no less than 87%) whether one observation is removed (*leave-one-out*) or all explanatory variables are orthogonalized. This stability argues against the hypothesis of a fortuitous fit driven by an outlier or by collinearity effects.

On the other hand, normality diagnostics for the residuals reveal a significant deviation, largely attributable to the year 2021, marked by an exceptional ex-post performance (+42.9%) linked to the anticipated exit from the COVID crisis. Removing this observation yields an ambiguous outcome: the Shapiro–Wilk test no longer rejects normality, whereas the D'Agostino–Pearson test remains marginally unfavorable.

Regarding autocorrelation, the Durbin–Watson test suggests a moderate risk, but Breusch–Godfrey tests at one and two lags do not confirm this finding. As a precaution, standard errors were adjusted using the Newey–West method with one and two lags. This correction slightly improves the significance of the coefficients without altering the hierarchy of explanatory variables, while maintaining a first-type error risk below 5% for all of them[22].

While the explanatory power of the present model should be interpreted with caution given the small size of the dataset, it nevertheless appears to provide promising evidence of the influence of the five selected variables on the market's ex-post return. These results complement and corroborate those we have previously obtained concerning the improvement of the predictive power of Shiller's CA–P/E ratio (see Clère, 2025). In particular, they suggest that stock market performance may be explained over a one-year horizon—something that statistical noise appeared to make unattainable with a CA–P/E–based approach. If confirmed by longer data series, these findings would indicate that the dynamics of stock market performance during the study period rested on a relatively stable valuation convention (which does not preclude variation in the influencing variables). Provided this convention remains in place, stock prices would then be largely predictable (i) from data known at the time of investment and (ii) over a shorter horizon than the ten years considered by Shiller (1998). Such a result would run counter to the semi-strong form of market efficiency formulated by Fama (1970), which posits that all available information is instantaneously reflected in prices, making returns impossible to forecast.

3.2. Inferring a Required IRR under Constant Uncertainty and Risk Conditions

Whether one seeks to explain the volatility of stock market returns or to estimate their expected value, it appears in both cases that central importance should be given to the rate of capital accumulation. As indicated in Equation (5), it is thus possible to decompose stock market performance $g\{P\}$ into a capital accumulation component, $g\{B\}$ and a valuation component $g\{P/B\}$. Growth would then consist of two components of distinct nature:

- The first component $g\{B\}$, reflects the growth of productive capacity, observed here through the lens of financial leverage. It is a macroeconomic variable of an objective nature, as it can be measured ex post.

[22] Furthermore, both the Breusch–Pagan and White tests show no evidence of heteroskedasticity in the residuals

- The second component, $g\{P/B\}$, is a valuation parameter (i.e., a multiple) that reflects expectations rather than objective ex-post growth. It is also more composite in nature than the first component, as it combines the expected return on equity, $E(ROE)$, the expected capital accumulation rate, $E(g\{B\})$ and the required return on equity, which we treat as the ex-ante IRR. Importantly, the first two parameters influence the estimation of the third, thereby highlighting the ambiguity of the causal relationship between price and the implied IRR—or conversely, between the required return and the price that results from it.

In any case, we have seen that simulating returns based on explanatory variables such as ROE, IRR, and capital growth likely makes it possible to reproduce both stock market performance and its volatility under conditions of constant uncertainty.

In a Brownian price diffusion model, the drift could plausibly be linked to capital accumulation, while the stochastic component would correspond to fluctuations in the valuation multiple.

Accordingly, under unchanged levels of uncertainty and price volatility, one could justify a given level of ex-ante required IRR, as suggested by Equation (11), derived from Appendix 2, section (c).

$$\mu + \frac{\sigma^2}{2} = ln(1 + IRR) \qquad 11)$$

Where:

- IRR refers to the required rate of return on equity investments, expressed as an annually compounded discrete rate.
- μ is the drift or central trend parameter in the Brownian diffusion process of returns. As discussed above, this parameter can arguably be approximated by the (logarithmic) capital accumulation rate, so that $g\{B\} \approx \mu$;
- and finally, σ^2, denotes the variance of logarithmic returns, which is an observable quantity. This variance could, it seems, be largely inferred from variables known at the time of investment, namely: i) the average ROE of listed firms, $\overline{ROE}$, ii) the forecasted growth in equity capital, g', which may be derived from economic growth projections (e.g., IMF forecasts), iii) and the IRRs estimated up to the investment date.

This could lead to a purely mechanical justification for a minimum level of equity returns—one that is not predicted by standard equilibrium models such as the CAPM. Indeed, in this framework, the IRR does not result from the sum of a risk-free rate and a risk premium, but instead arises directly from the recurrent volatility of the stock market and the rate of capital accumulation.

In this setting, the risk premium is derived from the IRR and the risk-free rate, whereas standard theory would instead derive the IRR from the sum of the risk-free rate and a risk premium justified in its own right. This causal inversion may help explain why the decline in the risk-free rate observed after the 2008 crisis did not lead to a corresponding decrease in the required IRR, as discussed in our previous work (Clère, 2025).

While this reasoning may be valid in describing a low-rate environment, there is no guarantee that the risk premium will not regain its assumed causal role in the event of a rate hike. In such a case, the IRR could exceed the level implied by capital accumulation and market volatility.

3.1. The Risk Premium as a Hedge Against Inflation

Equities are generally regarded as an asset class that, on average, provides a hedge for investment portfolio returns against inflation risk. Over the long run, this appears to have held true in the United States, whereas this principle was not always upheld in the case of T-Bonds, particularly during World War II. It is therefore possible that the determination of the equity risk premium obeys a rule not accounted for in the standard model. Investors may in fact require a risk premium not relative to the risk-free rate, but in addition to the observed inflation rate at the time of investment.

Over the long run, our previous work (Clère – 2025) compiled estimates of the implied internal rate of return (IRR) for the U.S. equity market from four sources—three academic (Jagannathan – 2000, Ke – 2014, and Mishra – 2019) and one private (Fairness Finance). When Mishra's estimates are adjusted for the average optimism bias arising from the use of analysts' forecasts—namely, a 2% premium (see Clère 2025, p. 12)—they appear comparable to those of Ke and Jagannathan, as well as to the net IRR estimated under the Fairness Finance model. This yields a series of net IRR estimates for the U.S. market, calculated as decade averages from the 1930s through the 2010s. These figures are presented in Table 5 below and compared with the Consumer Price Index (CPI) and with the yield on 20-year T-Bonds prior to 1970, and on 10-year T-Bonds thereafter (rf).

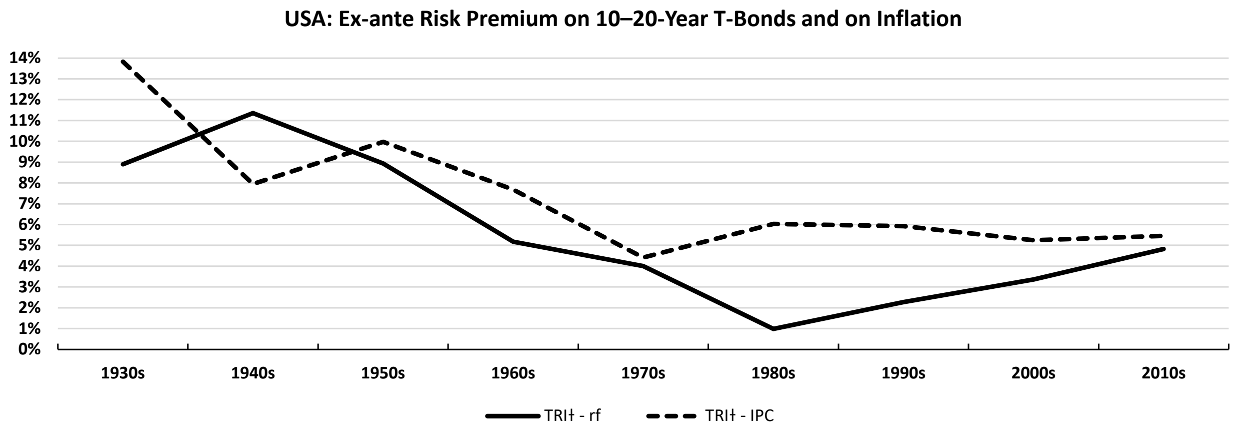
Table 5

Decade	Jagannathan	Ke	Mishra†	K+M†+FF†	Average*	rf	IRR† - rf	CPI	IRR†-CPI
1930s	11,9%				**11,9%**	3,0%	8,9%	-2,0%	13,8%
1940s	13,6%				**13,6%**	2,2%	11,4%	5,6%	8,0%
1950s	12,0%				**12,0%**	3,1%	8,9%	2,1%	10,0%
1960s	10,0%				**10,0%**	4,8%	5,2%	2,3%	7,7%
1970s	10,9%	12,1%			**11,5%**	7,5%	4,0%	7,1%	4,4%
1980s	11,3%	11,8%			**11,6%**	10,6%	1,0%	5,6%	6,0%
1990s	9,4%	8,8%	8,7%		**8,9%**	6,7%	2,3%	3,0%	5,9%
2000s		8,4%	7,3%		**7,8%**	4,5%	3,4%	2,6%	5,2%
2010s				7,2%	**7,2%**	2,4%	4,8%	1,8%	5,5%

* Average of Ke, Mishra, and Fairness Finance until 2012, followed by an estimate by Fairness Finance.
†The IRR is net of the premium for the optimism bias in cash flow forecasts.

As shown in Figure 6 below, the two series of average decennial risk premiums followed similar trends up until the 1970s, whether calculated relative to Treasury bond yields or to inflation. From the 1980s through the 2000s, however, this parallelism disappeared, giving rise to strongly positive real interest rates. In this context, equity IRRs no longer tracked the evolution of Treasury yields, and the risk premium relative to the latter collapsed during the Paul Volcker episode. By contrast, the gap between the net IRR and the inflation rate has remained relatively stable since that period, apparently fluctuating between 5% and 6%.

Figure 6



Over the past four decades, inflation appears to have become an explanatory variable for the required internal rate of return (IRR), alongside the growth rate of equity capital and stock market volatility—which itself is linked to the instability of economic growth. Once again, we observe that over the very long term, prior to the 1970s, a very different valuation convention prevailed. The 1930s thus appear as a window into a bygone world, marked by a heightened sense of uncertainty, likely inherited from the 19th century and the Industrial Revolution, whose influence gradually faded by the 1970s along with its last protagonists.

Looking ahead, a less globalized economy—very likely accompanied by a return of inflation—would tend to increase the cost of equity capital. A shift toward industrial reshoring could also help accelerate the pace of capital accumulation, $g\{B\}$, thereby amplifying the potential rise in the required IRR. The growing salience of uncertainty is not a new phenomenon; it is closely tied to the ongoing struggle for global leadership between the former hegemon, the United States, and China, or more broadly, the Global South. To this geopolitical dimension, one may have to add even more existential challenges, such as the potential exhaustion of cheap fossil energy and climate change. As a result, it seems unlikely that the current level of uncertainty will ease in any foreseeable future.

If we consider the likely possibility that uncertainty amplifies aleatory risk (i.e., stock market volatility), and given the very high sectoral concentration of the U.S. market, all indications suggest that achieving strong equity market performance in the future will be particularly challenging. This would likely require, at a minimum, an improvement in return on equity (ROE), combined with an initial decline in valuation multiples—such as the price-to-book ratio (P/B).

The American century—which probably extends from the victory in World War I to the present day—was marked by a virtuous cycle linking technological progress, mass consumption, and a decline in uncertainty. It is now worth asking whether the next phase will succeed in reconciling technological progress with the creation of sustainable wealth.

Conclusion

The models and regressions carried out indicate that it is possible to explain, at least partially, both stock market performance and its volatility. Over a one-year horizon, there is no conclusive evidence of intrinsic excess volatility arising from the functioning of the market itself—volatility that would represent an autonomous source of instability—as suggested by other studies adopting alternative approaches that do not incorporate the IRR and, consequently, the perception of uncertainty.

Secondly, this work offers two new convention-based explanations for the "equity risk puzzle." Over the past four decades, the required internal rate of return (IRR) appears to have been closely tied to the pace of capital accumulation and the volatility of the stock index, while also seemingly providing investors with protection against inflation. As a result, it would be determined largely independently of the risk-free rate. The equity risk premium would thus be a residual outcome, rather than a driving factor of the required return. Risk perception should therefore be sought less in the premium itself than in the parameters that directly determine the IRR and reflect both aleatory risk and fundamental uncertainty.

This work reveals the ambivalent manner in which risk is taken into account by the stock market. *Aleatory risk*, in Knight's sense—understood here as market volatility—may determine the required IRR when combined with both the rate of capital accumulation and inflation. However, this relationship has likely not always held in practice and therefore does not account for the long-term decline in IRRs observed throughout the 20th century. That trend seems instead to result from a changing perception of uncertainty. Even in the short term, shifts in that perception can disturb the relationship between volatility and returns, with uncertainty affecting both. These findings suggest that, although a theoretical distinction can be drawn between risk (as measurable randomness) and uncertainty, investors appear to blend the two in practice to form their perception of equity risk.

Finally, and in line with our previous work (Clère, 2025), stock market performance once again appears to be explainable using information available at the time of investment. This temporal precedence suggests a causal relationship and the operation of a form of procedural rationality, in which routines define a shared valuation convention that serves to navigate the fundamental uncertainty of the future. If such a convention proves relatively persistent, then—despite exogenous shocks and endogenous shifts in valuation parameters—it may be possible to anticipate stock market performance over a much shorter horizon than the one identified by Robert Shiller in his work on the CA-P/E ratio.

To confirm these initial empirical results, the dataset should be expanded with observations calculated at different dates and through the extension of the historical IRR series prior to 2014. Should these further investigations support our current interpretations, the semi-strong form of market efficiency would be called into question. This would imply that a substantial share of stock market performance is predictable— creating opportunities for portfolio managers capable of interpreting prevailing valuation conventions to pursue new investment strategies.

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Appendices

A1. Definition of the Volatility of Returns for a Stock or a Market Index

To begin, it is useful to briefly recall what is meant by the "return" of a financial asset (formerly referred to as its "yield") and the "volatility" of that return. To clarify the concepts, let us take the example of a stock. Its rate of return R_{t+1} observed between dates t and $t+1$ is defined as follows:

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} - 1 \qquad 6)$$

Where P_t and P_{t+1} denote the stock prices at the two successive dates, and D_{t+1} represents the dividend, if any, received by the shareholder between date *t* and the return calculation date, i.e., at $t+1$.

It can then be shown that this calculation, performed in *discrete time*, would be equivalent—under continuous compounding over infinitesimally small-time intervals, i.e., in *continuous time*—to the following logarithmic return, R^L_{t+1}:

$$R^L_{t+1} = ln(1 + R_{t+1}) = ln\left(\frac{P_{t+1} + D_{t+1}}{P_t}\right) \qquad 7)$$

This expression has the advantage—conferred by the logarithmic transformation—of replacing multiplication with addition. As a result, the continuously compounded return R_{t+1}^{L} can be treated as a random variable whose sum and average retain meaning in terms of interest compounding.

Accordingly, the volatility of the stock's return is defined here as the standard deviation of its returns observed at regular intervals—typically from one trading day to the next (at the same time), from month to month, or from year to year:

$$\sigma_{R^L} = \frac{1}{n}\sum_{i=1}^{n}(R_{t+i}^{L} - \bar{R}^{L})^2 \tag{8)}$$

Where σ_{R^L} denotes the standard deviation of the observed logarithmic returns, for which a statistical estimator may be computed where appropriate.

If returns are independent and identically distributed according to a normal distribution, then it can be shown that their variance is proportional to the frequency of the return calculation. In other words, a variance computed from weekly observations, when multiplied by 52 (i.e., the number of weeks in a year), yields a theoretical annual variance equivalent to one calculated directly from annual observations. Since variance is the square of the standard deviation, it follows that volatility is proportional to the square root of time:

$$\sigma_{anual}^2 = 52 \times \sigma_{weekly}^2 \iff \sigma_{anual} = \sqrt{52} \times \sigma_{weekly} \tag{9)}$$

It is generally at this point that the problem of unwarranted extrapolation arises, for example when one infers annual volatility from 30 weekly price observations to evaluate an option set to mature in 10 years. Yet, this is precisely how risk is often assessed over very distant horizons in many models used to value stock options granted to corporate executives (nowadays referred to as "preference shares" for tax and communication purposes). We return to the critique of this view of stock price movements as a random walk in the conclusion and keep in mind that this topic remains to be further investigated.

A2. Modeling the Theoretical Volatility of the U.S. Stock Market Using a Monte Carlo Simulation

The aim of this appendix is to outline the steps involved in the modeling process.

The first step in this process is to identify the data likely to be incorporated into the model to best reproduce the value of the stock index. These data must be reasonably assumed to be known by a large number of professional investors, who are presumed capable of using them to assess the valuation level of the index. In a second stage, the principles of the Monte Carlo simulation are presented, followed by its main results.

a) Market Data Selection

The first step of the modeling process involves identifying the market parameters that economic agents—such as portfolio managers and other professionals—are assumed to have at their disposal in order to apply the market index valuation model derived from Equation (2) in the main text, recalled below:

$$P_t = B_t + B_t \times \frac{\overline{ROE}_t - IRR_t}{TRI_t - g_t} \qquad 2)$$

As a former professional contributor to the estimation of implied returns (IRRs), I have had the opportunity to compare my results with those of peers within the *Société Française des Évaluateurs* (SFEV) and, through American clients who also subscribe to other data providers. Pending the release of a comprehensive public study, one may reasonably accept that the internal rates of return for listed equities, when estimated from analysts' forward-looking forecasts, are sufficiently convergent to allow for the emergence of a broadly shared market consensus. These market-level IRRs are derived from the aggregation of firm-level IRRs across the portfolio of listed stocks. Each of these individual IRRs is typically calculated using a valuation model akin to a discounted cash flow (DCF) framework. In this study, we rely on the IRR estimates provided by the *Fairness Finance* model, available from December 2014 onward for the U.S. market. The dataset compiled here extends through the end of March 2022. These IRRs are net of the optimism bias adjustment applied to forecasted cash flows (see below for details).

As shown in our previous work (Clère, 2025), the implied IRR appears to function as a leading explanatory variable for subsequent stock market performance. It shares this predictive quality with Robert Shiller's cyclically adjusted price-to-earnings ratio (CA-P/E). Should this finding be confirmed by further empirical evidence, it would suggest that the implied IRR reflects the prevailing valuation convention that governs equity pricing.

We therefore assume that the market's ex-ante implied rate of return (IRR) constitutes a key parameter within the prevailing valuation convention. This estimate is assumed to be known by professional investors and used both to assess the pricing level of the overall market and to evaluate individual securities. Once the aggregate market IRR is known, we proceed to identify the additional parameters required to implement equation (2). The index price P_t, l equity capital, and earnings—scaled to the market price—are provided by S&P and made available through several data providers. Specifically, index prices and earnings data can be accessed via Robert Shiller's public database, while aggregate book equity data can be retrieved from the website Mulpl.com.

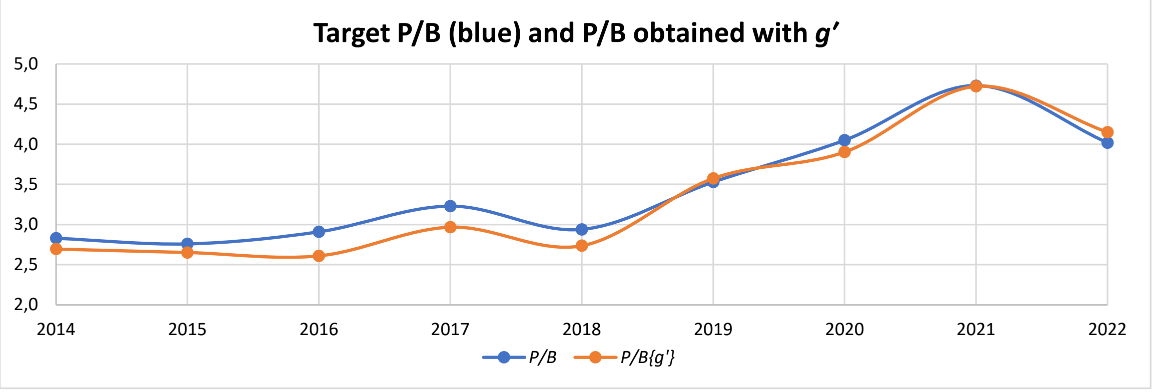
Based on the findings established in our previous work (Clère, 2025), we adopt a 10-year rolling average of annual ROEs as the normative estimate of $\overline{ROE}_t$. Accordingly, for the first year in the dataset (2014), the normative $\overline{ROE}_{2014}$ corresponds to the average ROE over the fiscal years 2005 through 2014.

Given that the price, book equity, and IRR are known, the remaining variable to estimate is the growth rate g_t. To identify the most suitable proxy, we rearrange equation (2) to express g_t as a function of the other parameters, then compare the resulting target values with available growth indicators in the databases. Upon review, the most appropriate approach appears to be pairing the net IRR with the IMF's 5-year forecast of nominal U.S. GDP growth. This macroeconomic projection serves as a consistent and widely used benchmark for anticipated capital accumulation:

- The net IRR is estimated by Fairness Finance based on analysts' forecasts, after adjusting for their optimism bias. This bias correction accounts for approximately 2 percentage points (200 basis points) in the IRR estimate. Thus, starting from a gross IRR estimate of 8%, the expected market return is adjusted downward to 6%.
- The IMF's forecast appears well suited to replicate the observed price-to-book (P/B) ratio. Indeed, the IRR estimated by Fairness Finance draws on the practices of market professionals in their DCF models. From this, an empirical rule has been derived: 75% of the IMF's nominal GDP forecast is used as the expected growth rate g' in equation (2). This is a market convention that, according to the model's authors, has remained stable over the past decade.

Accordingly, we adopt the pairing of the net IRR with a long-term nominal GDP growth estimate equal to 75% of the IMF forecast. The following graph compares the observed market price-to-book ratio with the reconstructed ratio obtained by adjusting the IMF's growth forecast (g'):

Figure 7



As can be seen, the use of this publicly available parameter appears to be highly appropriate for modeling the index's price levels.

b) Assumptions of the Monte Carlo Simulation

As stated in Equation (4) from the main body of the text, modeling the variation in the index price $g\{P\}$ involves modeling both (i) the growth of book equity $g\{B\}$ and (ii) the change in the price-to-book multiple $g\{P/B\}$:

$$ln\frac{P_t}{P_{t-1}} = ln\frac{B_t}{B_{t-1}} + ln\frac{P_t/B_t}{P_{t-1}/B_{t-1}} \qquad 4)$$

$$\Leftrightarrow \dot{P}/P = \dot{B}/B + \frac{\dot{P/B}}{P/B} \Leftrightarrow g\{P\} = g\{B\} + g\{P/B\}$$

b.1) Modeling the Growth of Book Equity

The variable $g\{B\}$ is assumed to follow a normal distribution, as confirmed by Shapiro–Wilk tests. Two alternative scenarios are considered:

- Under the optimistic and prudent scenario, we assume, $g\{B\} \sim N(5\ \%, 7.3\ \%)$;
- Under the pessimistic and myopic scenario, we assume, $g\{B\} \sim N(3{,}7\ \%, 2.9\ \%)$.

b.2) Modeling the Variation of the P/B Multiple

As stated in the main text, the variation in the price-to-book (P/B) multiple is assumed to have zero expected value. Recall the definition of the multiple:

$$\frac{P_t}{B_t} = 1 + \frac{\overline{ROE}_t - IRR_t}{IRR_t - g_t} \qquad 3)$$

The variables are estimated from data collected over the 2014–2022 window. They are all assumed to follow normal distributions, with Shapiro-Wilk tests confirming normality in all cases except for the growth rate variable g which nonetheless passes the D'Agostino-Pearson test at the 5% significance level. For simplicity, all variables are treated as normally distributed:

- $\overline{ROE} \sim N(13.8\ \%, 0.9\ \%)$,
- $TRI \sim N(6.4\ \%, 0.5\ \%)$,
- $g' \sim N(3\ \%, 0.14\ \%)$.

Moreover, the variables are correlated with each other:

Table 6

	g(B)	*µ(ROE)*	*IRR*	*g'*
g(B)	100%	-27%	15%	55%
µ(ROE)	-27%	100%	-55%	9%
TRI	15%	-55%	100%	-7%
g'	55%	9%	-7%	100%

It is also noteworthy that successive P/B ratios exhibit a first-order autocorrelation of 86%. To account for this phenomenon, we incorporate the observed autocorrelation coefficients of the explanatory variables between dates $t-1$ and t:

- $\rho(\overline{ROE}_{t-1,t}) = 71\ \%$;
- $\rho(TRI_{t-1,t}) = 56\ \%$;
- $\rho(g_{t-1,t}) = -8\ \%$.

c) Results of the Monte Carlo Simulation

Due to space constraints, we will limit our presentation to the results obtained under the "optimistic and prudent" scenario. A total of 20,000 random draws were conducted using *Crystal Ball*, an Excel-based Monte Carlo simulation software. This tool allows for the modeling of random variables while taking into account their correlations.

The key statistics of the simulated variables are summarized in the Table 7 below:

Table 7

Statistics	ln(Bt+1/Bt)	ln(P/Bt / P/Bt-1)	ln(Pt/Pt-1)	P/Bt	P/Bt-1	R
Draws	20 000	20 000	20 000	20 000	20 000	20 000
Mean	**5,03%**	**0,24%**	**5,28%**	**3,35**	**3,33**	**6,42%**
Median	5,04%	-0,07%	5,04%	3,22	3,22	5,17%
Mode	---	---	---	---	---	---
Standard deviation	**7,29%**	**10,24%**	**13,71%**	**0,81**	**0,73**	**14,78%**
Variance	0,53%	1,05%	1,88%	0,66	0,54	2,18%
Skewness	-0,01	0,17	0,11	1,29	1,16	0,55
Kurtosis	3,01	3,08	3,07	6,75	6,03	3,62
Coefficient of variation	1,45	42,00	2,60	0,24	0,22	2,30
Minimum	-26,18%	-36,93%	-46,66%	1,35	1,52	-37,29%
Maximum	38,79%	45,06%	66,08%	11,69	10,26	93,64%
Range	64,97%	81,99%	112,75%	10,34	8,74	130,93%
Standard error of the mean	0,052%	0,072%	0,097%	0,006	0,005	0,104%

An additional variable, R, was included in the simulation. It corresponds to the simple one-year return, defined as follows:

$$R = {}^{P_{t+1}}/_{P_t} - 1$$

The pairwise correlations between the simulated variables are as follows:

Table 8

	ln(Bt+1/Bt)	*n(P/Bt / P/Bt-1*	*ln(Pt/Pt-1)*	*P/Bt*	*P/Bt-1*	*R*
ln(Bt+1/Bt)	100%	20%	68%	12%	4%	68%
ln(P/Bt / P/Bt-1)	20%	100%	85%	39%	-5%	85%
ln(Pt/Pt-1)	68%	85%	100%	35%	-2%	100%
P/Bt	12%	39%	35%	100%	90%	36%
P/Bt-1	4%	-5%	-2%	90%	100%	-1%
R	68%	85%	100%	36%	-1%	100%

In theory, the expected value of the simple return, *E(R)*, is greater than the expected value of the logarithmic return that generates it, which is equal to the average of the log price changes: $\mu = E\left[ln\left({}^{P_{t+1}}/_{P_t}\right)\right]$, or $\bar{g}\{P\}$. This is the trend component, commonly referred to as the *drift*, which constitutes the deterministic part of a Brownian diffusion model for the random variable$P$. In this framework, returns are normally distributed: $g\{P\} \sim N(\mu \times \Delta t\,; \sigma \times \sqrt{\Delta t})$. The drift thus appears as the first term on the right-hand side of the equation below, with the second term representing the stochastic component:

$$g\{P\} = ln\left({}^{P_{t+\Delta t}}/_{P_t}\right) = \mu \times \Delta t + \sigma \times Z \times \sqrt{\Delta t} \quad | \quad Z \sim N(0{,}1) \qquad 10)$$

In this context, the time unit is one year, so the interval Δt equals one. The volatility σ is defined as the standard deviation of annual logarithmic price variations, i.e., $\sigma = \sigma(g\{P\})$.

Provided that logarithmic price variations follow a normal distribution, as stated in Equation (10) above, one can show that the return based on the expected future price, $\frac{E(P_{t+1})}{P_t}$, is derived from the drift μ :

$$\mu + \frac{\sigma^2}{2} = ln\frac{E(P_{t+1})}{P_t} \qquad 11)$$

From this, the expected value of the simple return, $E(R)$, can be derived as follows:

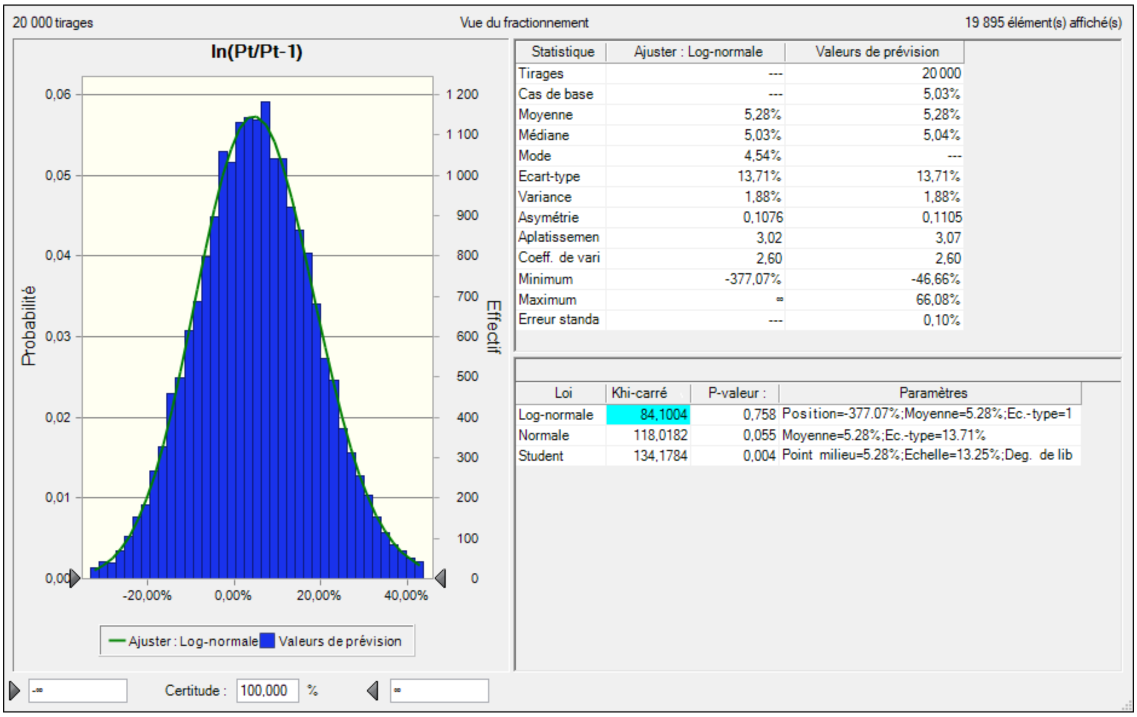
$$E(R) \; = \frac{E(P_{t+1})}{P_t} - 1 = e^{\mu + \frac{\sigma^2}{2}} \qquad 12)$$

If $g\{P\}$ indeed follows a normal distribution, then it follows that R, the simple return, is theoretically log-normally distributed (see Aftalion, 2008, ch. 1).

In the present case, the expected simple return $E(R)$ is equal to 6.42% (see the summary statistics Table 7 of the simulated variables). This simple return corresponds to a continuously compounded return of 6.22% (since $ln(1 + 6.42\%) = 6.22\%$), and implies a drift of 5.3%. These results are consistent with equations (11) and (12), given the observed average of the logarithmic returns, μ, and their variance σ^2.

As shown in the following graph (Figure 8), which presents the distribution histogram of the simulated returns μ he resulting distribution is slightly log-normal but very close to the expected normal distribution. This slight deviation is likely due to the correlation constraints imposed in the model. It does not significantly affect the estimate of $E(R)$, which remains close to 6.4%, consistent with the valuation convention for the period studied according to the Fairness Finance model.

Figure 8



Translation of statistical terms : Valeur de prévision (Forecast value), loi (Distribution), ajuster (Fit), Tirages (Draws), Cas de base (Base case), Moyenne (Mean), Médiane (Median), Écart-type (Standard deviation), Variance (Variance), Asymétrie (Skewness), Applatissement (Kurtosis), Coefficient de variation (Coefficient of variation), Minimum (Minimum), Maximum (Maximum), Erreur standard (Standard error), Lognormale (Lognormal), Normale (Normal), Point milieu (Midpoint), Échelle (Scale), Vue du fractionnement (Split view), Élément(s) affiché(s) (Displayed item(s)), Effectif / (Number of observations).