Paraconsistent logic for predicting S&P and Ibovespa futures prices

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Abstract

This study presents a decision-making model based on paraconsistent annotated evidential logic Eτ (PAEL-Eτ), applied to Ibovespa and S&P 500 futures trading. It identifies and resolves contradictions in technical analysis indicators, increasing investment decision reliability. Data from 1994 to 2023 train and test these models, applying moving average-based trading rules. Daily forecasts are generated from the recent past data, with a training and a testing period. Estimating degrees of certainty and uncertainty, PAEL-Eτ assesses the predictive reliability of each indicator. Results indicate that isolated indicators produce conflicting signals, confirming inherent inconsistencies. However, PAEL-Eτ mitigates these contradictions filtering unreliable signals. Despite short-term return improvements, statistical tests confirm no significant excess returns over buy-and-hold, reinforcing the weak-form efficient market hypothesis. The method provides an approach for handling contradictory, with potential applications in automated trading and hybrid AI models.

1 Introduction

The literature on financial asset price forecasting can be categorized into two main approaches. The first focuses on predicting the price levels of

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stocks, indices, and other financial instruments. Examples of this approach include the study by Sako et al. (2022), which forecasts the closing prices of stock market indices and exchange rates using recurrent neural networks (RNNs) and their variants. Additionally, Sobreiro et al. (2008) examine the prediction of sugar prices, while Pérez-Rodríguez et al. (2005) and Teixeira and Rodrigues (1997) compare various artificial intelligence-based forecasting methods, ultimately concluding that artificial neural network models yield superior performance in predicting the price levels.

The second class of studies focuses on forecasting the direction or signal of price level changes, as seen in the works of Chen et al. (2003), Chun and Kim (2004), and Souto-Maior et al. (2011), which emphasize the importance of index movement regardless of the observed magnitude. More recent studies, such as those by Patel et al. (2015), have further advanced this area by incorporating machine learning techniques like support vector machines and hybrid models to enhance predictive accuracy. Research on directional forecasting has gained increasing attention in recent years, as it provides valuable information while requiring relatively simple computational implementation.

Due to the characteristics of investors and analysts who rely on technical indicators, as well as their investment objectives, divergences in the interpretation of these indicators are often observed when forecasting market behavior Park and Irwin (2007). In an effort to address these interpretative divergences, three distinct evaluation approaches were employed: (i) the Traditional method, which follows conventional market practices for interpreting technical indicators; (ii) the Aggregated method, which combines multiple indicators to refine decision-making; and (iii) the Paraconsistent method, which applies the premises of paraconsistent annotated evidential logic Eτ (PAEL-Eτ) to overcome inconsistencies and contradictions between indicators.

Expert systems will be developed based on the three evaluation approaches. To achieve this, the effectiveness of the technical indicators obtained from each approach is analyzed within a training period, using a portion of the available dataset. This process helps identify the expert system that delivers the best performance and determines the optimal strategy for generating forecasts in the final period of the dataset.

Due to the contradictions frequently encountered in real-world scenarios—contradictions that classical logic is unable to handle—the idea for this study emerged, exploring an alternative class of logics: Paraconsistent Logic. This is a type of non-classical logic that accepts and processes contradictions in a non-trivial manner, allowing for intermediate logical states between absolute falsehood and absolute truth da Costa et al. (1991).

In this study, the problem lies in assessing the consistency of investment decisions based on forecasts generated from the most commonly used technical analysis indicators and their impact on trade management.

Initially, investment rules are applied to execute buy and sell operations based on technical analysis, using moving averages as the primary signaling indicator. The recommendations generated by these rules allow for an evaluation of results over time, identifying both scenarios in which decisions exhibit inconsistencies and those where they demonstrate greater coherence and alignment with the adopted strategy.

Next, the outcomes of these decisions are analyzed and compared through investment strategies, considering different management approaches based on both classical logic and paraconsistent logic.

With the support of paraconsistent logic, it is possible to identify inconsistent signals in the decision-making process and explore the best ways to address these situations. In this manner, contradictory signals regarding asset management can be systematically analyzed based on the degrees of certainty (μ_1) and uncertainty (μ_2) defined by paraconsistent logic ($\mu_1, \mu_2 \in [0, 1]$). This approach enables a more precise evaluation of investment viability, considering the level of requirement (N_{req}) involved in the analysis.

This study makes a novel contribution to the finance literature by demonstrating the advantages of combining technical analysis with PAEL-Eτ.

Next section presents a literature review on technical analysis and PAEL-Eτ, followed by the methodology for evaluating the Ibovespa and S&P 500 futures indices. Section four presents the results, while section five discusses final considerations and PS accuracy rates. The study concludes in the final section.

2 Literature review

2.1 Technical Analysis

According to Park and Irwin (2004), technical analysis is based on the implicit assumption that price movements are not entirely random but follow identifiable trends that can be exploited for forecasting purposes. This approach suggests that past price fluctuations contain valuable information that can be used to predict future market movements. Unlike fundamental analysis, technical analysis does not focus on the underlying economic, political, or financial factors influencing prices but rather on the patterns and trends that emerge from historical price data. The methodology assumes that these trends reflect the collective behavior of market participants, allowing traders to anticipate future price directions based on historical patterns.

Thus, technical analysis is based on the principle that all relevant information about an asset is already reflected in its price and trading volume. As a result, the technical analyst focuses solely on price movements without investigating their underlying causes.

Ratner and Leal (1999) analyzed moving average strategies in emerging markets and concluded that, after accounting for transaction costs, these strategies do not outperform the buy-and-hold approach. Park and Irwin (2007), in a comprehensive literature review, found mixed evidence regarding the effectiveness of technical analysis, highlighting methodological issues and a decline in profitability over time due to increasing market efficiency. On the other hand, more recent studies, such as that of Sermpinis et al. (2018), indicate that technical analysis still has short-term value in advanced, emerging, and frontier markets. Moreover, the performance of trading rules appears to be influenced by financial stress, the economic environment, and the level of market development. A cross-validation exercise conducted by the authors highlights the importance of frequent rebalancing and the variability of profitability in trading based on technical analysis.

The technical analyst frequently relies on technical indicators to support buy and sell decisions, such as moving averages, the Relative Strength Index (RSI), the Moving Average Convergence Divergence (MACD), among others. However, these indicators can generate conflicting signals: while one may suggest buying, another may indicate selling, and a third may recommend holding the asset. Given this, the development of an expert system that employs an internal decision-making process based on non-classical logics may be a useful approach to assist investors or analysts in making more informed decisions.

As an example, consider the signals generated by five moving averages, defined over different past periods, for the Ibovespa Futures Index on April 27, 2009. Trading rules based on short- and long-term moving averages indicated a buy signal for the combinations $S = 1$ and $L = 50$, $S = 1$ and $L = 150$, and $S = 5$ and $L = 150$, while the combinations $S = 1$ and $L = 200$ and $S = 2$ and $L = 200$ signaled a sell. Here, S represents the short-term window and L the long-term window in the moving average rule. Although this example does not provide the detailed calculations of the moving averages or graphs displaying the crossover points, it illustrates how different parameter configurations can lead to conflicting signals. Observing the index's movement after market close, a decline was recorded that day, suggesting that, despite three of the five rules indicating a buy, the opposite decision, sell, would have been the most accurate.

Given these results, the problem of contradictory signals generated by different moving average configurations arises, where the PAEL-Eτ approach is intended to be applied. This method estimates confidence levels for each generated signal by considering the past performance of moving averages within the historical sample. Paraconsistent logic enables the system to be trained with historical data, acknowledging that similar patterns can occur throughout the time series of the indices. The goal is not to disregard conflicting signals but to incorporate them into the analysis to generate a consolidated signal that takes all evaluated strategies into account.

The idea is to determine whether the current position in the asset should be maintained or whether it would be more appropriate to adjust the investment stance. Here, it is assumed that the Bovespa Futures and S&P 500 Futures are liquid markets, allowing investors to change their positions at any time.

2.2 Paraconsistent annotated evidential logic Eτ (PAEL-Eτ)

Paraconsistent logic belongs to the class of non-classical logics and originated independently in the late 1940s and early 1950s through the works of Stanislaw Jaśkowski, a Polish logician, and Newton C. A. da Costa, a Brazilian logician (Carvalho and Abe, 2011).

A logical system is said to be paraconsistent if it can serve as the underlying logic for paraconsistent theories—those that are inconsistent yet non-trivial da Costa (1980). This framework challenges the principle of non-contradiction, which states that, between two contradictory propositions, one must be false.

Paraconsistent logics were developed to provide a framework for handling contradictory situations, demonstrating their suitability for addressing problems that arise from contradictions in real-world descriptions Abe (1999).

According to these authors, inconsistencies in the real world typically emerge when multiple experts provide opinions on the same subject, leading to conflicting yet meaningful perspectives. Paraconsistent logic enables reasoning in such contexts without automatically collapsing into triviality, allowing contradictions to be analyzed rather than discarded[1].

In paraconsistent annotated logic, propositional formulas are accompanied by annotations[2]. Each annotation belongs to a finite lattice τ, which assigns a value to its corresponding propositional formula. The meaning of the proposition is understood through a formal language. In this logic, the annotation consists of two values: one representing the evidence in favor of proposition p, and the other representing the evidence against proposition p.

Intuitively, in paraconsistent annotated evidential logic Eτ (PAEL-Eτ), an annotation $(\mu_1; \mu_2)$ is assigned to each elementary proposition p, where μ_1 and μ_2 belong to the closed interval $[0, 1]$. In this framework, μ_1 represents the degree of belief (or favorable evidence) in p, while μ_2 represents the degree of disbelief (or contrary evidence), as described by Abe (1999)[3]. Thus, the propositional formula $p(\mu_1; \mu_2)$ can be informally interpreted as: "I believe in proposition p with a degree of belief μ_1 and a degree of disbelief μ_2."

2.2.1 Unit Square of the Cartesian Plane (USCP)

According to Abe (2015), the set of annotation constants $(\mu_1; \mu_2)$ can be represented in a Cartesian coordinate system by the unit square $[0; 1] \times [0; 1]$, referred to as the Unit Square of the Cartesian Plane (USCP), which represents the lattice τ, as shown in Figure 1. Each point composed of the pair $(\mu_1; \mu_2)$ within the USCP represents a logical state. The following extreme logical states are highlighted:

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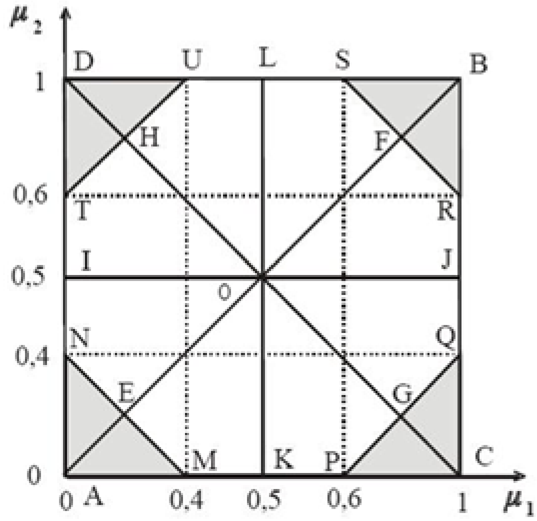
[1] A logical system is considered trivial when any contradiction within it leads to the derivation of any and all propositions, rendering the system incapable of distinguishing between true and false statements. A "non-trivial treatment" of contradictions means that inconsistencies are managed in a way that does not render the entire system logically useless.

[2] A statement or proposition constitutes a propositional formula when it is assigned one of two possible logical values: true or false.

[3] Unlike classical logic, which assigns only true or false values to propositions, paraconsistent annotated logic provides a more nuanced representation by associating annotations that quantify both belief and disbelief in a statement, allowing contradictions to be handled by recognizing that something can be partially true and partially false at the same time.

Figure 1: USCP divided into twelve regions, with limits $|G_{contr}| = 0.60$ and $|H_{cert}| = 0.60$. Source: Abe (1999).



1. $(1; 0)$ intuitively represents total belief and no disbelief (True);
2. $(0; 1)$ intuitively represents no belief and total disbelief (False);
3. $(1; 1)$ intuitively represents simultaneous total belief and total disbelief (Inconsistent $\top$);
4. $(0; 0)$ indicates the complete absence of both belief and disbelief (Paracomplete $\perp$).

2.2.2 Degrees of Contradiction and Certainty

The USCP can be divided in several different ways. One convenient division is into 12 regions, as shown in Figure 1[4].

Based on Carvalho et al. (2003) and Abe (2015), the following definitions are presented:

1. Degree of Contradiction: $G_{contr} = \mu_1 + \mu_2 - 1$ hence, $-1 \leqslant G_{contr} \leqslant 1$;

[4]The 12-region division balances decision granularity with practical applicability, ensuring meaningful interpretation without excessive complexity.

2. Degree of Certainty: $H_{cert} = \mu_1 - \mu_2$ hence, $-1 \leqslant H_{cert} \leqslant 1$.

The degree of contradiction (G_{contr}) and the degree of certainty (H_{cert}) are two complementary measures used in PAEL-Eτ to assess the logical state of a proposition based on the values of belief (μ_1) and disbelief (μ_2). The degree of contradiction, given by $G_{contr} = \mu_1 + \mu_2 - 1$ quantifies the extent to which belief and disbelief simultaneously exceed or fall short of a balanced state, with balance occurring along the first diagonal (A to B) in Figure 1, where belief and disbelief sum exactly to one. Meanwhile, the degree of certainty, defined as $H_{cert} = \mu_1 - \mu_2$ determines how much belief dominates over disbelief, placing a proposition along a certainty-falsehood scale. Geometrically, G_{contr} represents the deviation from the consistency axis, while H_{cert} indicates the deviation from the neutrality axis (D to C), in Figure 1, where belief equals disbelief. Together, these two measures provide a nuanced characterization of logical states, distinguishing contradictory, paracomplete, true, false, and indeterminate regions within the unit square representation of paraconsistent logic.

Given the above definitions, it is easy to see, from Figure 1, that:

1. MN: Paracompleteness boundary line, such that $G_{contr} = -k_1$, $0 < k_1 < 1$;

2. RS: Inconsistency boundary line, such that $G_{contr} = +k_1$, $0 < k_1 < 1$;

3. TU: Falsehood boundary line, such that $H_{cert} = -k_2$, $0 < k_2 < 1$;

4. PQ: Truth boundary line, such that $H_{cert} = +k_2$, $0 < k_2 < 1$.

It is customary to adopt $k_1 = k_2 = k$, providing symmetry to the graph, as shown in Figure 1, where $k_1 = k_2 = k = 0.60$. The value of k_2 will be referred to as the requirement level (N_{req}).

In Figure 1, four main extreme regions and a central region are highlighted:

1. AMN: Paracompleteness region, $-1 \leqslant G_{contr} \leqslant -0.60$;

2. BRS: Inconsistency region, $0.60 \leqslant G_{contr} \leqslant 1$;

3. CPQ: Truth region, $0.60 \leqslant H_{contr} \leqslant 1$;

4. DTU: Falsehood region, $-1 \leqslant H_{contr} \leqslant -0.60$.

The CPQ and DTU regions are referred to as decision regions. The first represents a favorable decision (viability), while the second corresponds to an unfavorable decision (non-viability).

In addition to the previously mentioned regions, we have the MNTUS-RQP region: $|G_{contr}| < 0.60$ and $|H_{cert}| < 0.60$, this is the region where decision-making is not possible. In other words, when the point representing the analysis result falls within this region, the analysis is considered inconclusive.

2.2.3 Decision Rule

If, for example, in the feasibility analysis of a business venture, the result falls within the CPQ region (truth region), the decision is favorable, meaning the venture is considered viable. Conversely, if the result falls within the DTU region (falsehood region), the decision is unfavorable, indicating the infeasibility of the venture. However, if the result falls in any region other than these two, the analysis is considered inconclusive.

These principles define the decision rule, as described by Carvalho et al. (2003), which can be summarized as follows: $H_{cert} \geqslant 0.60 \Rightarrow$ favorable decision (viability); $H_{cert} \leqslant -0.60 \Rightarrow$ unfavorable decision (infeasibility); $|H_{cert}| < 0.60 \Rightarrow$ inconclusive analysis.

Let us observe that $|H_{cert}| = 0.60$ was adopted as the boundary lines for truth and falsehood. This means that the analysis is only conclusive when $|H_{cert}| \geq 0.60$. Consequently, the value 0.60 (or 60%) represents the requirement level (N_{req}) of the analysis. Therefore, the requirement level determines the minimum value of $|H_{cert}|$ necessary for a result to fall within the truth or falsehood regions, that is, for a favorable or unfavorable decision to be made. This implies that decisions will only be made with at least 60% certainty.

In a more general form, the decision rule can be expressed as follows: $H_{cert} \geqslant N_{req} \Rightarrow$ favorable decision (viability); $H_{cert} \leqslant -N_{req} \Rightarrow$ unfavorable decision (infeasibility); $|H_{cert}| < N_{req} \Rightarrow$ inconclusive analysis.

The requirement level (N_{req}) depends on the degree of confidence desired in the decision, which, in turn, is influenced by factors such as the level of responsibility involved, the magnitude of the investment at stake, the presence of risk, and other contextual considerations.

It is also important to highlight that if the result lies within the BRS region (inconsistency region), the analysis remains inconclusive regarding the feasibility of the venture but indicates a high degree of data inconsistency ($G_{contr} \geqslant 0.60$). Similarly, if the result lies within the AMN region (paracompleteness region), it signifies that the data exhibit a high degree of indeterminacy ($G_{contr} \geqslant 0.60$).

2.2.4 Logical Operators in PAEL-Eτ

In PAEL-Eτ, logical operators are essential for handling uncertainty, contradiction, and incomplete information while maintaining logical coherence. Unlike classical logic, where propositions are strictly true or false, Eτ assigns degrees of belief (μ_1) and disbelief (μ_2), requiring adapted logical operations Abe (2015).

The NOT operator is defined as: $NOT(\mu_1; \mu_2) = (\mu_2; \mu_1)$. It inverts the evidential assessment, swapping belief and disbelief values while maintaining the structure of annotated logic. In this system, fundamental truth values transform as follows: $NOT(\top) = \top$, $NOT(\bot) = \bot$, $NOT(V) = F$, $NOT(F) = V$. Thus, the NOT operator ensures that logical negation is consistent with the evidential structure of PAEL-Eτ while allowing for contradictory and paracomplete evaluations to remain within the system.

The OR operator, analogous to classical disjunction, is defined as: $(\mu_1; \mu_2)$ $OR(\lambda_1; \lambda_2) = (\max\{\mu_1; \lambda_1\}; \max\{\mu_2; \lambda_2\})$. The strongest available evidence in favor of or against a proposition is preserved by this operation, ensuring that disjunction remains a maximization process in accordance with classical logic, but within the constraints of annotated logic. It is particularly useful in cases where different sources of information contribute varying levels of support and opposition to a given proposition.

The AND operator, which corresponds to classical conjunction, is defined as: $(\mu_1; \mu_2)AND(\lambda_1; \lambda_2) = (\min\{\mu_1; \lambda_1\}; \min\{\mu_2; \lambda_2\})$. This operator ensures that conjunction respects the most conservative estimates of belief and disbelief, preventing an overestimation of support or opposition when multiple sources of evidence are considered. By using a minimization process, the AND operator guarantees that the truth of a composite statement is not exaggerated by partial or conflicting information.

3 Methodology

In this study, we used price series from the futures contracts of stock market indices B3 (Ibovespa) and CME Group (S&P 500). The choice of futures indices is justified by the ease of executing short-selling orders on these contracts[5]. Currently, the Ibovespa futures contracts are traded on B3 under the ticker IND for the standard contract and WIN for the mini contract, while the S&P 500 futures contracts are traded on the CME Group under the ticker ES (E-mini S&P 500). The collected data consists of daily closing prices from July 1994 to December 2023, obtained from the Economática S.A. database.

For both Ibovespa and S&P 500 futures contracts, a $100,000 investment is simulated at the end of 2022 to evaluate the returns obtained from forecasts over the first six months and the entire year of 2023. This analysis is conducted based on each of the strategies defined in the methodology, for comparison against the buy-and-hold strategy. In both cases, different training periods are used with an overlapping dataset. The first training period includes price data from July 1994 to December 2022. The second training period considers only the year 2022 for each index. This approach aims to assess the model's adaptability and determine whether an extensive historical dataset is necessary for reliable forecasting.

At the end of the training period, the moving average rules are ranked based on their performance coefficient, considering the given sample, in order to establish each investment strategy.

3.1 Technical Analysis Indicators Applied

The moving average of order n of variable y at time t is given by:

$$MA(y)_{n,t} = \sum_{i=1}^{n} \frac{y_{t-i}}{n} \tag{1}$$

where y_t is the daily closing series of y at time t.

It is observed that $MA(y)_{n,t}$ provides an estimate for y_t, based on the past values of the series.

[5]It is also noteworthy that transaction costs were not considered. This assumption does not affect the obtained results. According to Saffi (2003) and MarketAxess (2020), for large institutional investors, such as investment funds, among others, transaction costs are close to zero.

Buy and sell rules at time t:

$$\begin{array}{l} buy \equiv MA(y)_{S,t} > MA(y)_{L,t} \\ sell \equiv MA(y)_{S,t} < MA(y)_{L,t} \end{array} \tag{2}$$

where $MA(y)_{S,t}$ is the short-term moving average S of y at time t, and $MA(y)_{L,t}$ is the long-term moving average L of y at time t.

Decision rule r for y with S and L at time t:

$$r(y)_{S,L,t} = \begin{cases} 1 & if \quad MA(y)_{S,t} > MA(y)_{L,t} \\ 0 & if \quad MA(y)_{S,t} < MA(y)_{L,t} \end{cases} \tag{3}$$

where the signals 0 and 1 indicate predictions for sell and buy, respectively.

Direction of the index y at time t.

$$d(y)_t = \begin{cases} 1 & if \quad y_t > y_{t-1} \\ 0 & if \quad y_t < y_{t-1} \end{cases} \tag{4}$$

Success in forecasting the index y at time t using rule r.

$$G(y)_{r,t} = \begin{cases} 1 & if \quad r(y)_{S,L,t} = d(y)_t \\ 0 & if \quad r(y)_{S,L,t} \neq d(y)_t \end{cases} \tag{5}$$

where 0 represents an error and 1 indicates a successful index forecast.

Performance coefficient in the prediction of y of order n for rule r at time t:

$$C(y)_{n,r,t} = \sum_{i=1}^{n} \frac{G(y)_{r,t-i}}{n} \tag{6}$$

which measures the number of times rule r correctly predicts the direction of the movement of the index y, expressed as a percentage for each period.

Five decision rules will be analyzed based on the signals predicted by $r(y)_{1,50,t}$, $r(y)_{1,150,t}$, $r(y)_{1,200,t}$, $r(y)_{5,150,t}$ and $r(y)_{2,200,t}$, following the approach of Ratner and Leal (1999), which employs similar indices and the same buy and sell rule.

Based on these signals, three independent strategies will be considered to determine the decision-making process, aiming for a comparison with the buy-and-hold strategy. These are: Traditional Strategy (TS), Aggregated Strategy (AS), and Paraconsistent Strategy (PS), as defined below.

Traditional Strategy (TS): The decision is made based on the rule that exhibits the highest performance coefficient $C(y)_{n,r,t}$ over the entire training period, where n represents the number of data points available in the training sample.

Aggregated Strategy (AS) for y at time t:

$$\mathrm{AS}(y)_t = \begin{cases} 1 \ if \ \dfrac{r(y)_{1,50,t} + r(y)_{1,150,t} + r(y)_{1,200,t} + r(y)_{5,150,t} + r(y)_{2,200,t}}{5} > 0.5 \\ 0 \ if \ \dfrac{r(y)_{1,50,t} + r(y)_{1,150,t} + r(y)_{1,200,t} + r(y)_{5,150,t} + r(y)_{2,200,t}}{5} < 0.5 \end{cases} \tag{7}$$

again, 0 for sell and 1 for buy, thus forming the decision based on the majority signal, considering all defined rules.

Paraconsistent Strategy (PS): After determining the signals for index positions within the training period, the final decision is made while also considering contradictory information. However, this requires following specific steps in the application of paraconsistent annotated logic. These steps are presented below.

1st) Determine which rules will have greater importance in the final decision.

This will be based on the value of the performance coefficient $C(y)_{n,r,t}$ within each training sample.

2nd) Establish the level of scrutiny.

In this case, the level will be gradually increased by 10% starting from 60%, which is commonly used in the application of paraconsistent logic, until reaching 100%. This approach tests multiple levels, beginning with the minimum acceptable threshold, thereby restricting the acceptability region. This process defines the para-analyzer device and the decision rule.

3rd) Annotation of degrees of certainty and uncertainty.

$\mu_{1r,t}(y)$ and $\mu_{2r,t}(y)$ represents the degree of certainty and uncertainty, respectively, of the statement given by rule r at time t.

$\mu_{1r,t}(y)$ is defined as $C(y)_{5,r,t}(y)$, which uses short-term movements of the index, where $\mu_{1r,t}(y)$ corresponds to its weekly performance coefficient. For each rule individually, the degrees of certainty and uncertainty are considered complementary, meaning: $\mu_{1r,t}(y) + \mu_{2r,t}(y) = 1$.

4th) Obtain the resulting degrees of certainty $\mu_{1r,t}(y)$ and uncertainty $\mu_{2r,t}(y)$.

After each rule determines its degree of certainty $\mu_{1r,t}(y)$ and uncertainty $\mu_{1r,t}(y)$, the resulting degrees are obtained by applying the maximization (OR) and minimization (AND) techniques of paraconsistent annotated logic. Here, the OR maximization operator is applied internally to the group formed by the two lowest performance coefficients within the sample. The *AND* minimization operator is then applied between groups, meaning between the three highest performance coefficients and the result of the maximization process from the two lowest.

5th) Using these values to reach the final decision by calculating the resulting degree of certainty ($H_{certR,t} = \mu_{1R,t} - \mu_{2R,t}$).

To assess whether the return of the expert system-based strategy is statistically higher than that of the buy-and-hold strategy, we apply a t-test. Before conducting the test, an F-test can be used to verify whether the population variances are equal. If they are, the pooled variance estimator is applied, and the degrees of freedom are adjusted to $n_1 + n_2 - 2$. If they are not, Welch's t-test is used instead Montgomery and Runger (2010).

The return of index y between t and $t-1$ is defined as:

$$ret(y)_t = \frac{y_t - y_{t-1}}{y_{t-1}} \tag{8}$$

To evaluate the performance of a given strategy x relative to a benchmark index y, it is employed the generalized Sharpe index (GSI), given by:

$$\text{GSI} = \frac{\overline{ret(x)} - \overline{ret(y)}}{s(\overline{ret(x)} - \overline{ret(y)})} \tag{9}$$

where $ret(x)$ and $ret(y)$ represent the average returns of the strategy and the benchmark index over a given time period. The index y serves as the benchmark for comparison.

The intuitive interpretation of the GSI follows from its structure: it measures the excess return of strategy x over the benchmark y, standardized by the volatility of this excess return. This standardization allows for a risk-adjusted comparison between different strategies. A higher GSI value indicates that strategy x generates higher returns than the benchmark, given the observed variability in performance. The GSI is particularly useful in financial analysis because it provides a way to assess whether an investment strategy adds value beyond the market performance while accounting for fluctuations in returns. This metric is widely used in portfolio management to determine whether the additional risk taken by a strategy is justified by its returns.

Table 1: Accuracy of the rules according to $C(y)_{n,r,t}$

Future	Trainning	Rules				
		1-50	1-150	1-200	5-150	2-200
Ibovespa	1st (1994-2022)	0.5096	0.5097	0.5079	0.5143	0.5103
	2nd (2022)	0.5200	0.4200	0.4400	0.5000	0.4400
S&P500	1st (1994-2022)	0.5092	0.5196	0.5270	0.5263	0.5277
	2nd (2022)	0.4264	0.5147	0.5147	0.5000	0.5147

Note: Ibovespa first and second training included 6990 and 249 days, respectively. S&P500 first and second training included 7324 and 264 days, respectively. Forecast of 2023 for Ibovespa and S&P500 envolved 248 and 261 days respectively.

4 Results

The signaling provided by the indicators yields quite similar results whether the training period for Ibovespa spans from July 4, 1994, to December 29, 2022 (6990 days) or is limited to only the year 2022 (250 days) and for S&P500 futures spans from July 4, 1994, to December 30, 2022 (7324 days) or is limited to only the year 2022 (266 days), suggesting that an extensive dataset may not be necessary for applying the model. It is worth noting that the forecasting and testing period covered January 2, 2023, to December 29, 2023 (248 days for Ibovespa and 261 days for S&P500).

4.1 Ibovespa Future Index

4.1.1 First Trainning (1994 to 2022)

According to the adopted traditional strategy (TS), Table 1 shows that the $r(y)_{5,150,t}$ achieved the highest accuracy coefficient within the training period and was therefore applied throughout the entire year of 2023. Consequently, forecasts were made based on its signals. The evolution of the capital using this rule's predictions is illustrated in Figure 2. The top section of Table 2 presents the results for the application of the capital over six months and one year.

Initially, it is observed that just between March 20 and March 29, 2023,

Table 2: Forecasting results for the Ibovespa Future in 2023

Trainning	Test period	TS	AS	PS 60%	PS 70%	PS 90%
	Six-month					
1st	$\mu(\%)$	-0.0566	-0.0888	0.1072	0.0888	0.0888
(1994-	σ	1.2725	1.2707	1.2692	1.2707	1.2707
2022)	β	-0.1950	-0.2180	0.8327	0.9564	0.9564
	GSI	-0.0640	-0.0796	0.0520	0.0527	0.0527
	t-test	-0.7792	-0.9788	0.2355	0.1213	0.1213
2nd	$\mu(\%)$	-0.0542	-0.0888	0.1437	0.1004	0.1004
(2022)	σ	1.2726	1.2707	1.2656	1.2698	1.2698
	β	-0.2506	-0.2180	0.8086	0.9611	0.9611
	GSI	-0.0613	-0.0796	0.0959	0.0898	0.0898
	t-test	-0.7642	-0.9788	0.4624	0.1931	0.1931
	One year					
1st	$\mu(\%)$	0.0104	-0.0056	0.1159	0.1087	0.1061
(1994-	σ	1.1556	1.1557	1.1498	1.1505	1.1508
2022)	β	0.2580	0.2442	0.8751	0.9690	0.9696
	GSI	-0.0548	-0.0656	0.0497	0.0760	0.0672
	t-test	-0.7441	-0.8991	0.2748	0.2055	0.1803
2nd	$\mu(\%)$	-0.0134	-0.0056	0.1342	0.1145	0.1119
(2022)	σ	1.1556	1.1557	1.1478	1.1500	1.1502
	β	-0.1162	0.2442	0.8602	0.9718	0.9724
	GSI	-0.0585	-0.0656	0.0776	0.1026	0.0934
	t-test	-0.9742	-0.8991	0.4518	0.2616	0.2364

The values of Buy-and-Hold for six-month test períod are $\mu = 0.0692$ and $\sigma = 1.2719$, and for one year test period are $\mu = 0.0875$ and $\sigma = 1.1523$.

the traditional strategy (TS) achieves a higher average return than the passive strategy (buy-and-hold), at the cost of a slightly higher standard deviation (σ), reflecting increased risk. Additionally, the strategy exhibits a negative beta (β), indicating an opposite to market movements. However, over the one-year testing period, the average return of the TS falls below that of the passive strategy, suggesting that its superior performance in the shorter term may not have been sustained over a longer horizon.

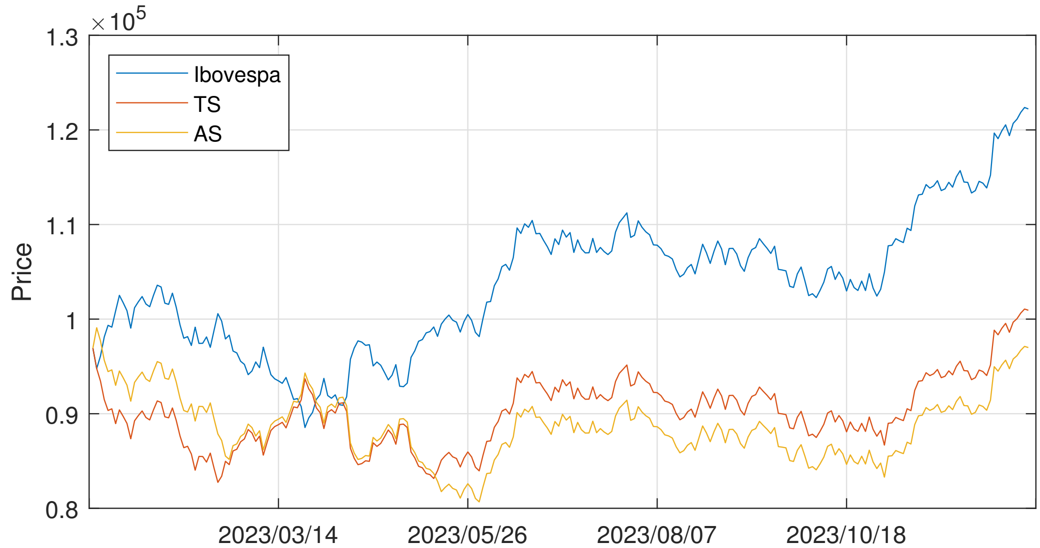
The t-test (0.05) was conducted to compare the returns of the buy-and-hold strategy and the traditional strategy (TS), with the null hypothesis assuming equal returns. The test results failed to reject the null hypothesis, indicating that the Traditional Strategy did not generate returns superior to the passive market strategy over either the six-month or one-year period.

The aggregated strategy (AS), which was defined based on majority signaling, produced signals similar to those of the traditional strategy (TS), indicating that the rule $r(y)_{5,150,t}$ largely aligns with the overall indications derived from all signals, except for a few isolated instances. It is also observed that AS disregards some signals, as the decisions of the minority are ignored. The forecasts based on this rule are shown in Figure 2, while Table 2 presents the results for the application of the capital over six months and one year. Over the six-month testing horizon, the results are similar to those of the traditional strategy (TS), the correct signal of aggregated strategy (AS) for second day forecast was the difference between those strategies, that turned AS superior until May, 16th, 2023. After that TS remains superior over six-month and one year test periods. This is confirmed by examining the signals during the testing period, as the majority signaling follows the signals of the 5-day and 150-day moving average rule for short- and long-term periods, respectively.

The Aggregated Strategy (AS) exhibits the same implications after May, 16th, 2023 as the traditional strategy (TS) over the six-month testing period and over the one-year period, the average return of the AS is lower than that of the buy-and-hold strategy and the TS. The results indicate a slight improvement for the AS compared to the TS, as it presents a lower standard deviation (σ) for the six-month, but a lower beta (β), although both still below one.

The t-test (0.05) was conducted to compare the returns of the buy-and-hold strategy and the AS, with the null hypothesis assuming equal returns. The null hypothesis was not rejected, yielding a result similar to that of the TS, indicating that the AS did not generate returns superior to the passive

Figure 2: Forecast using TS and AS vs buy-and-hold for Ibovespa Future in 2023.



market strategy in either testing period.

In the paraconsistent strategy (PS) defined here, the initial level of requirement was set at 60% and progressively increased by 10% increments until reaching 100%, which characterizes the viability of the decision predicted by a given rule as being considered true.

The buy and sell operations based on each of the rules analyzed here follow the principle of investing by conclusive signals and remains passive when the signals are inconclusive. If non-viability is established, the investor exits the index (sells), ensuring that capital is not maintained in an asset unviable. Thus, whenever there is a signal confirming viability, the strategy is executed accordingly.

The degree of certainty ($\mu_{1r,t}$) and uncertainty ($\mu_{2r,t}$), presented in Table 3, were determined at each time period in two ways to generate the signals: using the average between $C(y)_{5,r,t}$ and $C(y)_{20,r,t}$, and using only $C(y)_{5,r,t}$, focusing exclusively on very short-term signals without considering their monthly performance.

For both approaches, groups A, B, C, and D were formed, as the grouping

Table 3: Coefficients $\mu_{1r,t}$ and $\mu_{2r,t}$ for the groups and the resulting value

Future	Trainning	Groups	A	B	C	D	Final Output
Ibovespa	1st (1994-2022)	$\mu_{1r,t}$	0.4	0.2	0.2	0.6	0.2
		$\mu_{2r,t}$	0.6	0.8	0.8	1	0.6
	2nd (2022)	$\mu_{1r,t}$	0.6	0.4	0.2	0.2	0.2
		$\mu_{2r,t}$	0.4	0.6	0.8	1	0.4
S&P500	1st (1994-2022)	$\mu_{1r,t}$	0.6	0.6	0.6	0.6	0.6
		$\mu_{2r,t}$	0.4	0.4	0.4	0.4	0.4
	2nd (2022)	$\mu_{1r,t}$	0.6	0.6	0.6	0.6	0.6
		$\mu_{2r,t}$	0.4	0.4	0.4	0.4	0.4

Results for the first forecasted day.

is based on the entire historical dataset reserved for training the series.

Thus, the groups are structured as follows: Group A includes the rule $r(y)_{5,150,t}$, which has the highest accuracy coefficient; Group B consists of the rule $r(y)_{2,200,t}$, which has the second-highest coefficient; Group C contains the rule $r(y)_{1,150,t}$; and Group D comprises the rules $r(y)_{1,50,t}$ and $r(y)_{1,200,t}$, which have the two lowest accuracy coefficients. Within Group D, an intraclass maximization technique is applied using the OR operator from Paraconsistent Annotated Logic to optimize decision-making within the group. After this internal maximization process, the AND operator is applied across Groups A, B, C, and the result from Group D, ensuring a structured and refined final decision.

As an example of the application of PAEL-Eτ, Table 3 presents the resulting certainty and uncertainty values for the first forecasted day in the testing period. The D column in Table 3 illustrates how certainty and uncertainty values may not always be complementary, highlighting a key feature of the paraconsistent logic framework.

Using only its weekly performance $C(y)_{5,r,t}$, the model demonstrates good adherence to the data. The number of viability signals is relatively small compared to the total number of forecasts, and furthermore, this number tends to decrease significantly and stabilize as the level of requirement increases.

Table 4: Accuracy of conclusive signals based on PAEL-Eτ

Future	Trainning	Req. Level				
		60%	70%	80%	90%	100%
Ibovespa						
	1st	0.5964	0.7142	0.7142	0.7000	0.7000
	2nd	0.6415	0.7692	0.7692	0.7777	0.7777
S&P500						
	1st	0.5061	0.3636	0.3636	0.4444	0.4285
	2nd	0.5125	0.3636	0.3636	0.4444	0.4285

However, the observed results following conclusive signaling improve as the requirement level (N_{req}) increases for the Ibovespa in second trainning, as shown in Table 4. At a 60% requirement level, the accuracy rate of the conclusive signal was 64.15%. When the requirement level was raised to 70%, this accuracy increased to 76.92% and remained constant at 80%. Once the requirement level exceeded 90%, the accuracy rate stabilized at 77.77%. Figure 3 illustrates the evolution of the capital under the strategy for $N_{req} = 60\%$, $N_{req} = 70\%$ and $N_{req} = 90\%$.

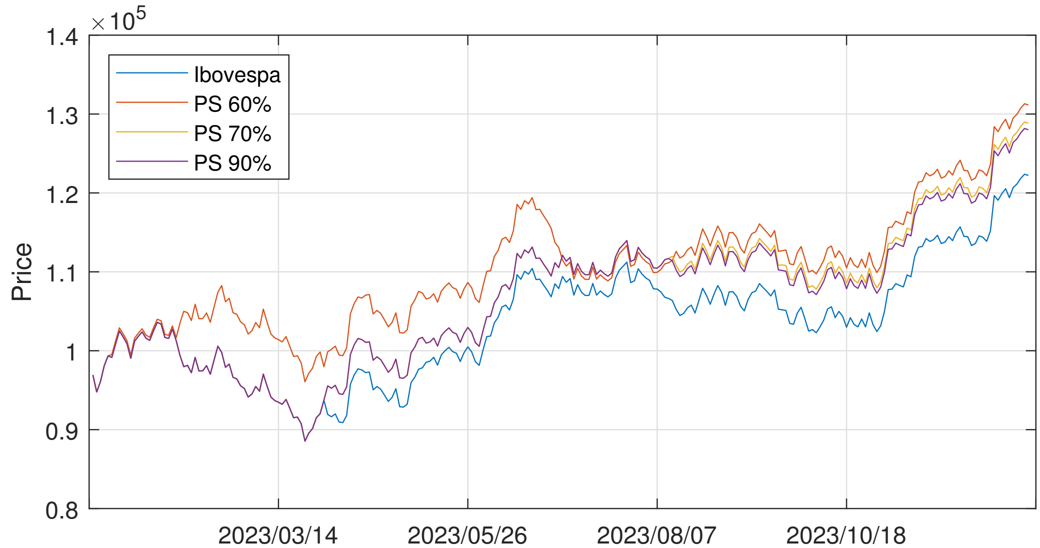
However, these results still fall short of generating returns above the market, as the number of viability signals is very small compared to the total number of forecasts made.

Since the signals for requirement levels of 90% or higher are the same, their comparison graphs with the buy-and-hold strategy are identical and are represented in Figure 3.

Table 2 presents the results for PS in the two test periods. Over a six-month horizon, at both the 60% and 70% requirement levels, the average return is lower than that of the buy-and-hold strategy. Consequently, the value of σ is also lower at both requirement levels. Additionally, it is observed that as the requirement level increases, the β value of the strategy also rises, approaching one.

For a one-year test period, it seems that the average returns increase considerably compared to the previously analyzed strategies, greater than passive strategy. Not only the return values but also the β values are now closer to one.

Figure 3: Forecast using PS at 60%, 70% and 90% requirement vs buy-and-hold for Ibovespa Future in 2023.



However, in the t-test (0.05) conducted for the difference between the returns of the paraconsistent strategy at 60%, 70% and 90% and the buy-and-hold strategy, the null hypothesis that the returns were equal was not rejected. This result is analogous to that of the traditional strategy, indicating that there are no returns above the market passive strategy through the paraconsistent strategy at either 60%, 70% or 90% in both test periods.

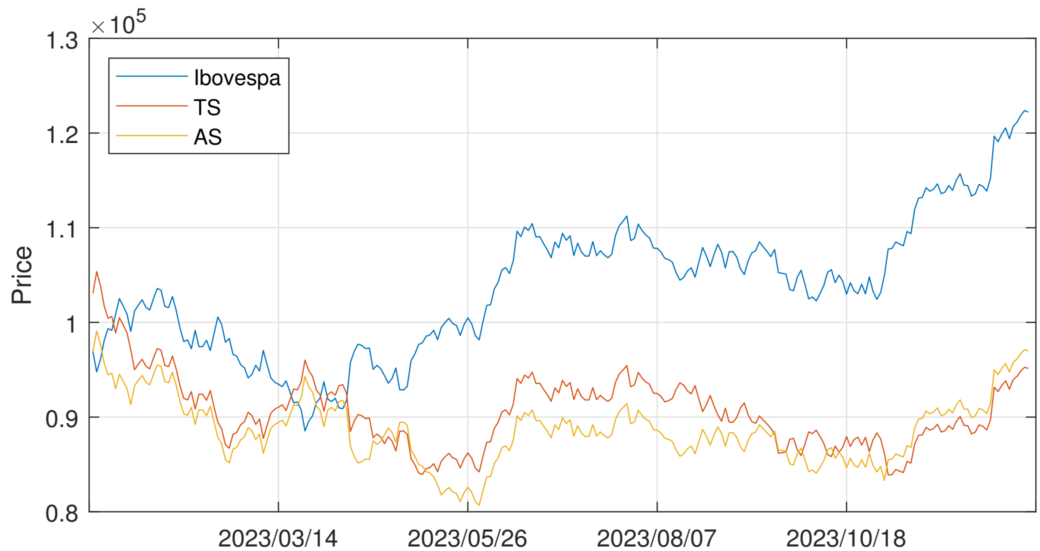
The presence of market efficiency in its weak form can be observed across all methodologies for the Ibovespa future index in this training, making it possible, but not significant, to achieve above-market gains based on past values of the series.

4.1.2 Second Trainning (2022)

For the traditional strategy (TS), the rules that achieved the highest efficiency coefficient within the training period were the rule $r(y)_{1,50,t}$, except between April, 26 and May 9 that was $r(y)_{5,150,t}$, as their signals were similar during the period. Therefore, forecasts for the entire year of 2023 were made using their signals. Figure 4 presents the financial evolution based on the forecasts of this strategy, and Table 2 shows the results obtained for the two test periods. For this strategy, there are small changes in the results compared to those found for the same strategy in the first training.

For the aggregated strategy (AS), which was defined by the majority signal, the signals were similar to TS, showing that the rules $r(y)_{5,150,t}$ and $r(y)_{1,150,t}$ align with the indications derived from all signals, except for isolated moments where the aggregated signal differs from $r(y)_{1,50,t}$ and $r(y)_{5,150,t}$. Although this strategy incorporates all rules, it still disregards some signals, as the minority decision is ignored. The evolution of the amount based on the forecasts from this rule is shown in Figure 4. Table 2 presents the results for the application of the amount over the two test horizons.

Figure 4: Forecast using TS and AS vs buy-and-hold for Ibovespa Future in 2023.



For the paraconsistent strategy (PS), the initial requirement level was set at 60% and was gradually increased in successive increments of 10% up to 100%, at which point a decision predicted by a given rule is considered viable and deemed true.

Operations according to each of the rules analyzed here follow the principle of investing when conclusive signal is confirmed. If viability is not confirmed, the investor remains passive if there are inconclusive signal. Thus, when there is a conclusive signal the operation follows its direction, either buy (viable) or sell (not-viable).

The degree of certainty ($\mu_{1r,t}$) and uncertainty ($\mu_{2r,t}$) were determined in each time period in two ways to generate signals: using the average between $C(y)_{5,r,t}$ and $C(y)_{20,r,t}$, and using only $C(y)_{5,r,t}$, focusing solely on very short-term signals without considering their monthly performance.

For both approaches, groups A, B, C, and D were formed, as the grouping is based on the entire historical data reserved for training the series.

Thus, the groups are as follows: Group A, with rule $r(y)_{1,50,t}$, which have the highest efficiency coefficient; Group B, with rule $r(y)_{5,150,t}$, which has the second-highest coefficient; Group C, with rule $r(y)_{2,200,t}$; and Group D, which uses rules $r(y)_{1,150,t}$ and $r(y)_{1,200,t}$, as they present the two lowest coefficients. These groups are used for the intra-group application of the maximization technique through the OR operator of annotated paraconsistent logic. After applying the internal maximization technique to Group D, the AND minimization technique is applied among Groups A, B, C, and D.

As an example, it is observed that the values of the certainty and uncertainty degrees resulting from the first forecasted day in this training are similar to those from the previous training. Figure 5 illustrates the evolution of this strategy.

These results may provide above-market returns, but the number of conclusive signals is small compared to the total number of forecasts made. Moreover, some signals deemed conclusive may incorrectly predict the index's direction.

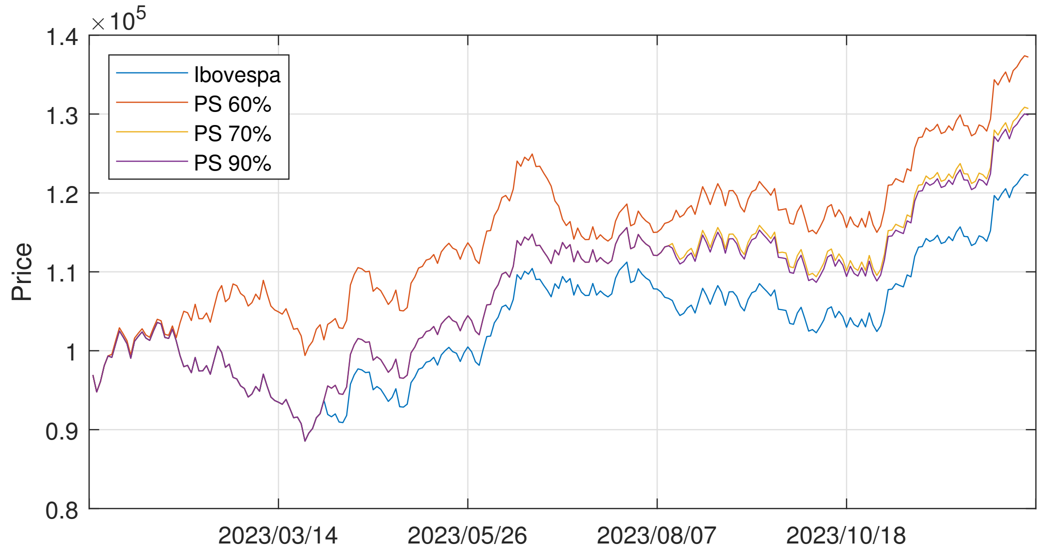
Since the signals for levels of 70% and 80%, 90% or higher are the same, their graphs, when compared to the buy-and-hold strategy, are identical and are represented in Figure 5. Table 4 further confirms these similarities for the paraconsistent strategy.

4.2 S&P500 Future

4.2.1 First Trainning (1994 to 2022)

According to the TS of this methodology, the rule that achieved the highest efficiency coefficient within the first training period was rule $r(y)_{2,200,t}$. The first test signal will be based on this rule, and as results are updated throughout 2023, the efficiency coefficients are recalculated, allowing the rule that

Figure 5: Forecast using PS at 60%, 70% and 90% requirement vs buy-and-hold for Ibovespa Future in 2023.



defines the strategy's decision to be adjusted. Throughout 2023, forecasts were made using the signals from $r(y)_{2,200,t}$ in both test periods.

This strategy appears to be in line with the efficient market theory (weak form), which states that it is not possible to achieve above-market returns based on past data. This can be observed in Figure 6, which illustrates the evolution of the application over one year, and in Table 5, which presents the results.

Table 5 shows that over the six-month and one year test periods, the traditional and aggregate strategies yields a lower average return than the passive strategy. The σ for six months is higher than that of the passive strategy, inconsistent with its lower average return. A β value smaller than 1 and a negative GSI indicate that the strategy has a poor performance than the passive strategy.

The t-test (0.05) was conducted to analyze the difference between the returns of the passive and traditional strategies. The null hypothesis assumed that the returns were equal, and it was accepted, indicating that the traditional strategy, on average, does not yield different returns compared to the

Table 5: Forecasting results for the S&P500 Futures in 2023

	Test period	TS	AS	PS 60%	PS 70%	PS 90%
	Six- month					
1st	$\mu(\%)$	0.0020	0.0243	0.0922	0.0954	0.1137
Trainning	σ	0.8816	0.8813	0.8767	0.8764	0.8742
(1994-	β	0.6281	0.5816	0.7865	0.9689	0.9952
2022)	GSI	-0.1529	-0.1168	-0.0460	-0.1019	-0.0543
	t-test	-1.0758	-0.8704	-0.2451	-0.2158	-0.0460
2nd	$\mu(\%)$	0.0414	0.0243	0.0922	0.0954	0.1137
Trainning	σ	0.8806	0.8813	0.8767	0.8764	0.8742
(2022)	β	0.6107	0.5816	0.7865	0.9689	0.9952
	GSI	-0.0992	-0.1168	-0.0460	-0.1019	-0.0543
	t-test	-0.7132	-0.8704	-0.2451	-0.2158	-0.0460
	One year					
1st	$\mu(\%)$	0.0125	0.0099	0.0778	0.0751	0.0857
Trainning	σ	0.8114	0.8115	0.8078	0.8080	0.8070
(1994-	β	0.6552	0.6030	0.7144	0.9810	0.9971
2022)	GSI	-0.1121	-0.1080	-0.0170	-0.0803	-0.0385
	t-test	-1.0696	-1.1051	-0.1471	-0.1861	-0.0352
2nd	$\mu(\%)$	0.0204	0.0099	0.0799	0.0751	0.0857
Trainning	σ	0.8113	0.8115	0.8076	0.8080	0.8070
(2022)	β	0.6299	0.6030	0.7150	0.9810	0.9971
	GSI	-0.0969	-0.1080	-0.0136	-0.0803	-0.0385
	t-test	-0.9577	-1.1051	-0.1181	-0.1861	-0.0352

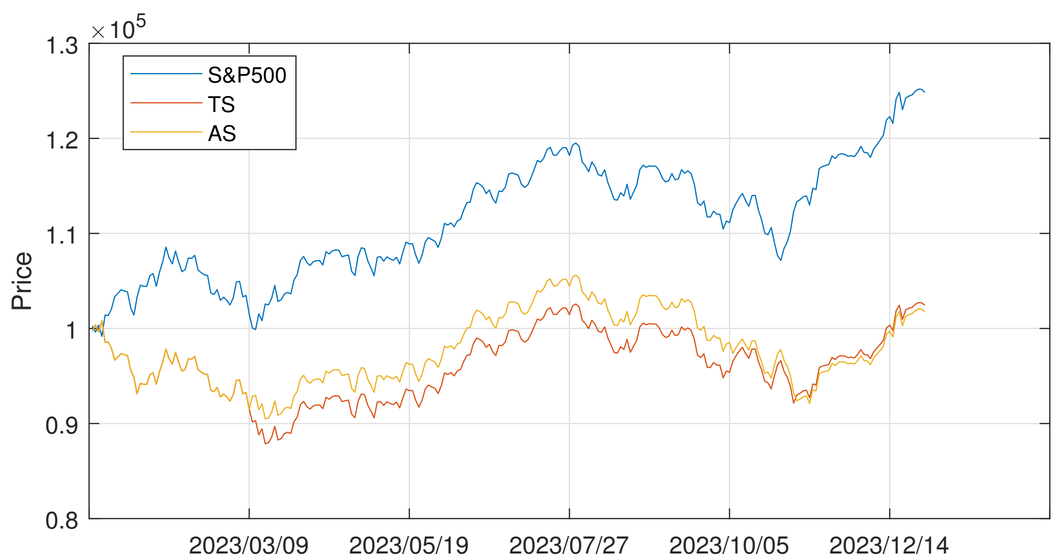
The values of Buy and Hold for six-month test períod are $\mu = 0.1187$ and $\sigma = 0.8735$, and for one year test period are $\mu = 0.0882$ and $\sigma = 0.8067$.

passive strategy, whether over six-month or one year test periods.

In the AS, the signal obtained from the majority indication showed a better return than that observed in the TS in the six-month test period, but worse in the one year test period, demonstrating that a decision based on the signals from all rules may lead to greater gains compared to a single specific rule in short term.

As defined in this study, although AS incorporates all rules, it ultimately disregards some signals, as it ignores the decisions of the minority, which may occasionally provide correct indications. Figure 6 illustrates the financial evolution based on decisions made under this strategy, while Table 5 presents the corresponding results.

Figure 6: Forecast using TS and AS vs buy-and-hold for S&P500 Futures in 2023.



The results show that over a six-month and one year investment horizon, the average return is lower than that of the passive strategy, with higher value of σ in six-month and smaller in one year than the passive strategy. A β positive is an indicator for the strategy's profitability follows passive strategy.

Once again, a t-test (0.05) was conducted to compare the returns of the aggregated strategy and the passive strategy. The null hypothesis assumed that the returns were equal, and it was not rejected, indicating that the aggregated strategy, on average, does not generate different returns compared to the passive strategy in either test period.

For the paraconsistent strategy (PS), the requirement level starts at 60% and increases successively by 10% until reaching 100%, determining whether the decision predicted by the rule is considered true.

The degrees of certainty ($\mu_{1r,t}$) and uncertainty ($\mu_{2r,t}$) are determined in each time period using two approaches to generate signals: one based on the average between $C(y)_{5,r,t}$ and $C(y)_{20,r,t}$, and another using only $C(y)_{5,r,t}$, focusing solely on very short-term signals without considering their monthly performance.

For both approaches, groups A, B, C, and D were formed, as the grouping is based on the entire historical data reserved for training the series.

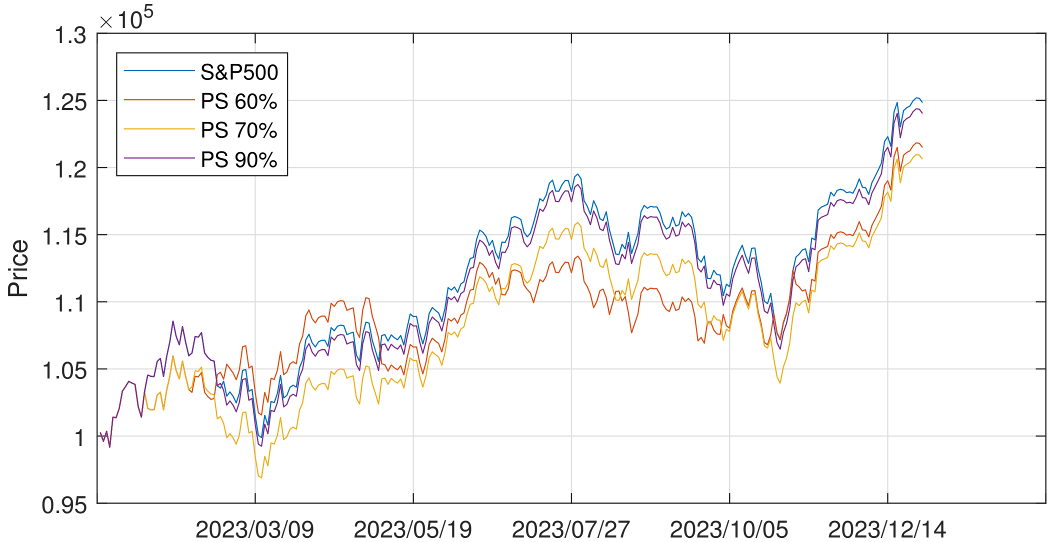
The groups are as follows: Group A, with rule $r(y)_{2,200,t}$, which has the highest efficiency coefficient; Group B, with rule $r(y)_{1,200,t}$, which has the second-highest coefficient; Group C, with rule $r(y)_{5,150,t}$, and Group D, which uses rules $r(y)_{1,50,t}$ and $r(y)_{1,150,t}$, as they have the two lowest efficiency coefficients. These groups are used for the intra-group application of the maximization technique through the OR operator of annotated paraconsistent logic. After applying the internal maximization technique to Group D, the AND minimization technique is applied among Groups A, B, C, and D.

Similarly, the model demonstrates good adherence to the data. The number of signals indicating viability is small compared to the total number of forecasts made, but it is now higher when considering performance defined by the average between weekly and monthly data. Furthermore, according to previous results, this number tends to decrease substantially and eventually stabilize.

Here, the results observed after the viability signals differ from those of the Brazilian market. Starting at a 60% requirement level, the accuracy rate for viability, initially 54.54%, decreases to 42.85% when the requirement level is raised to 70% and remains at this accuracy rate as the requirement level continues to increase (see Table 6).

However, higher requirement levels are associated with better returns for this strategy. The highest return observed was achieved with a 90% requirement level. At 60% as this setting generated 44 viability signals, of which 24 were successful, leading to returns close to those of the market. At 70% or higher requirement level, even with a few viable signals and lower accuracy the correct signals at 90% requirement level leads to a better performance (see Figure 7). For higher requirement levels, the average returns were higher

Figure 7: Forecast using PS at 60%, 70% and 90% requirement vs buy-and-hold for S&P500 Futures in 2023.



than those obtained at the 60% level, except for 70% in the one year test period. The charts represent the evolution of the application for the 60%, 70% and 80% levels, which share the same signals, and similarly for the 90% and 100% levels. These are shown in Figure 7. Table 5 presents the results for both requirement levels and for the two test periods.

For a six-month period, at a 60% and 70% requirement level, the average return is lower than that of the passive strategy, and the value of σ is higher than that of the passive strategy. Additionally, it is observed that as the requirement level increases, the strategy's β value also rises, approaching 1.

For a one-year test period, average returns increase considerably compared to the previously analyzed strategies, at the 90% approaching to passive strategy and equaling at 100% requirement level. When the requirement level increases σ value decrease and approaches of the passive strategy, β is very close to but slightly below one. Additionally, attention is drawn to the negative but increasing GSI values.

The t-test (0.05) conducted to compare the returns of the paraconsistent strategy at 60%, 70% and 90% with those of the buy-and-hold strategy assumed a null hypothesis that the returns were equal, which was not rejected. This result is analogous to those of the traditional and aggregated strategies, indicating that, on average, there are no different returns compared to the passive strategy.

Through this training, it was confirmed that it is not possible to sustain returns above those of a passive strategy, verifying the presence of weak-form market efficiency for the S&P 500 Futures index as well. However, the evolution of the applied amount, maintained under the signals of the PS with a 60% requirement level, showed a return superior to that of the passive strategy between February 21th and May 4th.

4.2.2 Second Trainning (2022)

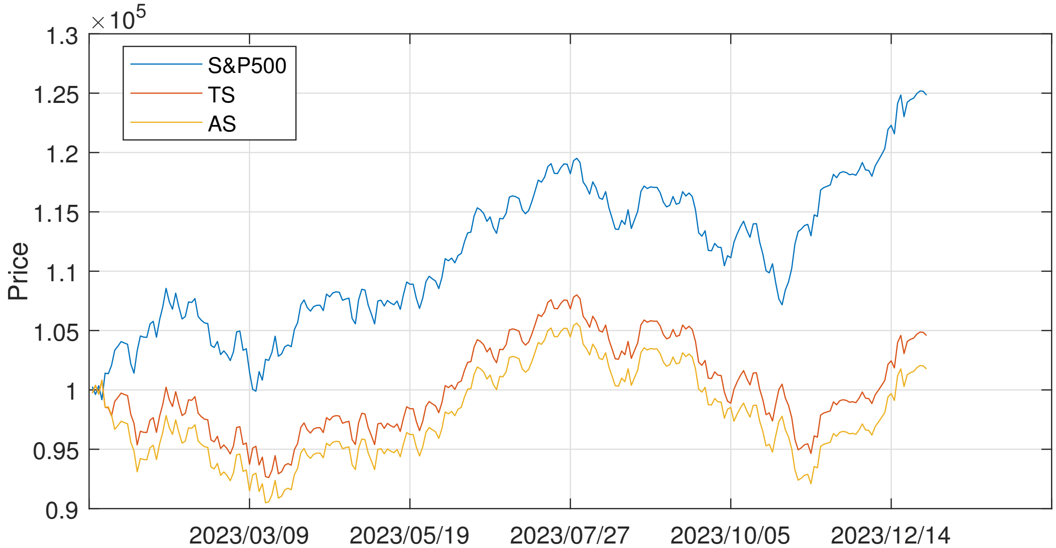
In the second training, the traditional strategy (TS) started the testing period with the signals of the rules $r(y)_{1,150,t}$, $r(y)_{1,200,t}$ and $r(y)_{2,200,t}$ that presented the same performance coefficient throughout the training period and the same signal from the first forecast day until 01/06/2023. With the error of the rule $r(y)_{1,150,t}$ on 01/09/2023, the performance coefficient of the rules r(1-200) and $r(y)_{2,200,t}$ became higher, so the signal for 01/10/2023 was given by these rules and was incorrect.

However, the sign of rule $r(y)_{1,150,t}$ on 10/01/2023 was correct, which implied the change of coefficients, leading rule $r(y)_{1,150,t}$ to remain as the rule with the highest performance coefficient for the remainder of the test period, except between 05/10/2023 and 20/10/2023, that the highest performance coefficient was $r(y)_{1,200,t}$. Figure 8 shows the financial evolution of the traditional strategy (TS) in the second training and the superior performance of this strategy compared to the aggregated strategy (AS).

According to Table 5, for the six-month test period, the average return was 0.041% compared to 0.002% in the first training and for the one-year test period, 0.020% compared to 0.012% in the first training. In both cases, σ and β were lower, but GSI was higher even negative.

For the case of the paraconsistent strategy (PS) at 60%, there was a small difference at the end of the one-year testing period. Table 5 illustrates that in the six-month test period the financial evolution was the same, which implies identical statistics, while for the one-year test period there was a subtle increase in the average return, in β and GSI with a small reduction in

Figure 8: Forecast using TS and AS vs buy-and-hold for S&P500 Futures in 2023.



the value of σ, reflecting an incorrect signal in the first training for 11/7/2023 (see Figure 9).

5 Final considerations and PS accuracy rates

This study examined the predictive accuracy of three different forecasting strategies—traditional strategy (TS), aggregated strategy (AS), and paraconsistent strategy (PS)—for the Ibovespa and S&P 500 futures markets in 2023. While no single strategy consistently outperformed the market, the paraconsistent strategy (PS) demonstrated improved accuracy at higher requirement levels, suggesting its potential to enhance decision-making in uncertain environments. While TS and AS generated daily predictions based on all available market data, PS applied a logical filtering mechanism to consider only signals with sufficient consistency. This methodological distinction significantly affected not only the frequency and accuracy of predictions but also the cumulative transaction costs, which were substantially lower for PS given its reduced trading frequency.

Figure 9: Forecast using PS at 60%, 70% and 90% requirement vs buy-and-hold for S&P500 Futures in 2023.

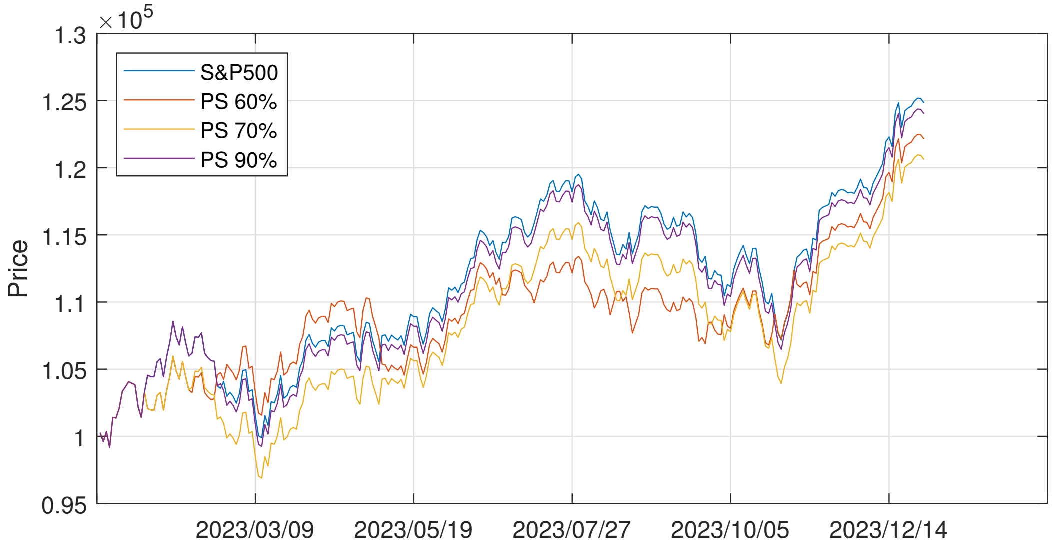


Table 6 summarizes the accuracy rates and statistical significance levels for each strategy. While TS and AS maintained consistent predictive frequencies across all trading days, PS exhibited selective decision-making based on different levels of requirement. As expected, higher requirement levels reduced the number of viable trading signals but increased accuracy rates. Notably, for the Ibovespa market, PS surpassed TS and AS at requirement levels of 70% and above, likely due to its ability to filter out inconsistent signals, focusing only on high-certainty predictions. This suggests that a stricter selection criterion enhances predictive accuracy by reducing noise from conflicting market signals. Moreover, the lower number of trades executed by PS translates into significant savings on transaction costs compared to TS and AS.

The results presented in Table 6 suggest that the selective approach of PS enhances predictive accuracy, particularly at higher requirement levels, albeit at the cost of fewer trading opportunities. From a practical standpoint, the reduction in transaction costs further strengthens the viability of PS in real-world applications. The statistical significance remains inconclusive due to

Table 6: Predictive Accuracy for Each Strategy (Number of Correct Predictions / Total Viable Cases)

Strategy	Ibovespa	S&P500
TS	122 / 248	134 / 261
AS	119 / 248	135 / 261
PS at 60%	19 / 36	24 / 44
PS at 70%	4 / 7	3 / 7
PS at 80%	4 / 7	3 / 7
PS at 90%	3 / 5	3 / 7
PS at 100%	3 / 5	3 / 7

Note: Statistical significance was assessed using Fisher's exact test (two-tailed), testing against the null hypothesis of 50% predictive accuracy. None of the strategies achieved statistical significance at the 5% level. Therefore, p-values are omitted from the table for clarity.

the limited number of viable cases, but the trend indicates potential benefits of using logical filters in financial decision-making. Future research should explore larger datasets and alternative parameterizations of PS to validate these preliminary findings further.

6 Conclusion

This study examined the application of paraconsistent annotated evidential logic Eτ (PAEL-Eτ) as a decision-support tool for financial markets, particularly in managing inconsistencies in technical indicators. The study applied three distinct forecasting strategies—traditional, aggregated, and paraconsistent—across historical data from Ibovespa and S&P 500 futures from 1994 to 2023 to assess their effectiveness compared to the passive buy-and-hold strategy.

The traditional strategy (TS) and aggregated strategy (AS), which relied on selecting the best-performing rule and combining multiple signals, respectively, showed some short-term predictive advantage but failed to maintain higher returns over longer periods. The paraconsistent strategy (PS), incorporating PAEL-Eτ to handle contradictory signals, demonstrated more stable returns, particularly at a 60% requirement level, where average returns exceeded those of the buy-and-hold strategy. However, t-tests confirmed that these excess returns were not statistically significant, reinforcing the weak-form Efficient Market Hypothesis (EMH) and suggesting that technical analysis alone may not provide a sustainable edge in financial markets.

Despite these findings, PAEL-Eτ proves valuable in refining decision-making processes by systematically handling conflicting signals, offering a structured framework for interpreting contradictory market indicators. The results suggest that future research should explore its integration with machine learning techniques to enhance predictive accuracy. Additionally, alternative trading environments, such as high-frequency markets or cryptocurrency exchanges, could provide new insights into the effectiveness of paraconsistent logic in financial decision-making.

In short, although this study does not find strong evidence of consistently outperforming the market, it highlights the value of PAEL-Eτ as a powerful tool for enhancing investment decisions in uncertain and contradictory scenarios. By addressing inconsistencies in market signals, this approach opens the door to more refined applications in financial forecasting, offering investors a structured way to navigate complex market dynamics.

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