Forecasting Value-at-Risk for Cryptocurrencies

May 25, 2025

Abstract

Value-at-Risk (VaR), the primary measure of downside risk in market risk management, relies heavily on the accuracy of volatility forecasts produced by risk models. This paper shows that, for forecasting the VaR of cryptocurrencies, the time-heterogeneous Student's t autoregressive model outperforms standard models commonly used by practitioners.

Key words: cryptocurrencies; financial risk forecasting; market risk; risk forecast models; Student's t autoregressive models; VaR

JEL: C18, C51, C52, C53, C58, G17, G32

1 INTRODUCTION

Cryptocurrencies have surged in popularity in recent years, attracting attention not only from individual investors but also from institutional investors. According to CoinDesk, several major U.S. university endowment funds have been buying cryptocurrencies (Allison, 2021), signaling a shift in institutional investor behavior. This trend extends to traditional financial institutions as well. In a significant move, the Basel Committee on Banking Supervision (BCBS) proposed prudential standards that would enable banks to hold certain types of crypto-assets on their balance sheets, subject to strict capital and risk management requirements (BCBS, 2021). The expanding institutional adoption of cryptocurrencies underscores their evolving role in investment portfolios, potentially contributing to increased portfolio volatility.

Previous studies have shown that inaccurate VaR forecasts often arise from model risk (e.g., Boucher et al., 2014). Among the various sources of model risk identified in the finance literature (see references in Danielsson et al., 2016), a particularly common source is "model choice": inappropriate assumptions about the form of the statistical model (Alexander and Sarabia, 2012). In general, model risk tends to increase with portfolio volatility, as higher volatility typically necessitates more complex statistical assumptions to accurately forecast VaR. As a result, standard risk models often fail to deliver reliable VaR forecasts for highly volatile portfolios (Michaelides and Poudyal, 2024) or during periods of financial crisis (Danielsson et al., 2016). Given the unique and often extreme risk characteristics of cryptocurrencies relative to traditional assets, their growing presence in portfolios is likely to further complicate accurate VaR forecasting.

1.1 Literature on VaR forecasting for cryptocurrencies

Given that cryptocurrencies are relatively new assets, few studies have compared risk models for forecasting their VaR. Troster et al. (2019) performed a general generalized autoregressive conditional heteroskedasticity (GARCH) and generalized autoregressive score (GAS) analysis for modeling and forecasting Bitcoin returns and risk. They found that heavy-tailed GAS models provide the best conditional and unconditional coverage for VaR forecasts. Liu et al. (2020) tested whether VaR of Bitcoin, Litecoin, and Ethereum can be forecasted using exponentially weighted moving average (EWMA) models, similar to the RiskMetrics approach of J.P. Morgan. Their results show that VaR can be successfully forecasted with parsimonious EWMA models, with the Laplace GAS specification, which controls for time variation in scale and skewness parameters, performing best at most confidence levels.

Jiang et al. (2022) applied their proposed time-varying mixture-accelerating GAS (TVM-aGAS) model to VaR forecasting of Bitcoin, XRP, and Litecoin. They showed that the TVM-aGAS model performs better compared to other standard models. Panagiotidis et al. (2022) performed a large-scale analysis to evaluate the performance of 27 alternative GARCH models for forecasting VaR of 292 cryptocurrencies. Their results indicate that time-varying models outperform traditional ones. Alexander and Dakos (2023) investigated the relative performance of different types of EWMA models and various GARCH models for forecasting VaR of Bitcoin, Ethereum, XRP, and Litecoin. Their findings demonstrate that simpler models in the EWMA class are just as accurate as GARCH models for VaR forecasting, provided they capture an asymmetric volatility response and a heavy-tailed distribution.

1.2 Aim of the paper

The primary aim of this paper is to evaluate the effectiveness of the time-heterogeneous Student's t autoregressive (t-StAR) model, proposed by Michaelides and Poudyal (2024), in forecasting VaR for cryptocurrencies. While the model has demonstrated strong performance in forecasting VaR for traditional assets such as equities and currencies, its applicability to cryptocurrencies remains untested. This study examines whether the t-StAR model can deliver reliable VaR forecasts in this rapidly evolving and highly volatile asset class.

Cryptocurrencies differ fundamentally from traditional assets due to unique characteristics such as decentralization, continuous 24/7 trading, and intense speculative activity. These attributes contribute to abrupt price swings, challenging the statistical assumptions that underlie many standard risk forecast models. Consequently, the suitability of models originally developed for traditional assets becomes questionable when applied to cryptocurrencies, where such assumptions are often violated.

The t-StAR model incorporates several features identified in prior research (see references above) as crucial for mitigating model risk in VaR forecasting, particularly in highly volatile settings. Specifically, it explicitly accounts for a heavy-tailed distribution and accommodates a conditional variance that is both heteroskedastic and time-heterogeneous. This paper evaluates whether these statistical assumptions enable the t-StAR model to accurately capture the volatility dynamics of cryptocurrencies, benchmarking its forecasting performance against risk models commonly used in practice.

In addition to contributing to the broader literature on financial risk forecasting, this paper offers valuable insights for practitioners (e.g., risk managers) responsible for managing portfolio risks, regulators (e.g., the Federal Reserve System) tasked with ensuring financial stability, and standard setters (e.g., the BCBS) who develop guidelines that shape risk management practices. By presenting empirical evidence of the t-StAR model's superior performance in forecasting the VaR of cryptocurrencies, the paper makes a strong case for more robust market risk assessment practices, particularly for portfolios exposed to high volatility. Furthermore, the model's improved forecast accuracy has implications for regulatory frameworks, such as enhancing alignment with Basel capital adequacy requirements. Collectively, these insights contribute to the development of more effective market risk management strategies that address the unique challenges posed by cryptocurrencies.

2 DATA

2.1 Sample of cryptocurrencies

The data for this paper were collected from CoinMarketCap[1], a widely used source of cryptocurrency price information. We initially considered the top 20 cryptocurrencies by market capitalization (see Table 1), but most were excluded due to insufficient historical data, as many had been launched only recently. We therefore focused on cryptocurrencies with several years of data, resulting in a final sample of Bitcoin (BTC)[2], Ethereum (ETH), XRP (XRP), Dogecoin (DOGE), and Litecoin (LTC).[3] The sample period for each cryptocurrency begins with its earliest available observation (see Table 1) and ends on May 31, 2022.

Table 1. Top 20 cryptocurrencies by market capitalization
The table lists the top 20 cryptocurrencies by market capitalization, along with their respective launch years and the earliest available price observations. Data were obtained from CoinMarketCap. Cryptocurrencies shown in bold are those included in the analysis.

#	Cryptocurrency	Launch year	Earliest observation	#	Cryptocurrency	Launch year	Earliest observation
1	**Bitcoin (BTC)**	2009	4/28/2013	11	Polkadot (DOT)	2020	8/20/2020
2	**Ethereum (ETH)**	2015	8/7/2015	12	Wrapped Bitcoin (WBTC)	2019	1/30/2019
3	Tether (USDT)	2014	2/25/2015	13	TRON (TRX)	2017	9/13/2017
4	USD Coin (USDC)	2018	10/8/2018	14	Avalanche (AVAX)	2018	9/22/2020
5	BNB (BNB)	2017	7/25/2017	15	Dai (DAI)	2017	11/22/2019
6	Cardano (ADA)	2017	10/1/2017	16	Shiba Inu (SHIB)	2020	8/1/2020
7	**XRP (XRP)**	2012	8/4/2013	17	Polygon (MATIC)	2017	4/28/2019
8	Binance USD (BUSD)	2019	9/20/2019	18	UNUS SED LEO (LEO)	2019	5/21/2019
9	Solana (SOL)	2020	4/10/2020	19	Cronos (CRO)	2021	12/14/2018
10	**Dogecoin (DOGE)**	2013	12/15/2013	20	**Litecoin (LTC)**	2011	4/28/2013

2.2 Descriptive statistics

To measure the relative price changes of the cryptocurrencies over time, daily closing prices are converted into log returns. Table 2 presents basic descriptive statistics for

[1]Available at: https://coinmarketcap.com (Accessed June 1, 2022).

[2]An exception occurred on April 15, 2022, when Bitcoin (BTC) price data were unavailable on CoinMarketCap. In this case, the corresponding value was sourced from Yahoo! Finance (https://finance.yahoo.com, accessed June 1, 2022).

[3]Tether (USDT), a stablecoin, was excluded despite having sufficient data, as its value is pegged to the U.S. dollar and thus exhibits minimal price volatility.

each cryptocurrency. As expected, all return series exhibit high standard deviations and excess kurtosis, indicating significant volatility and heavy-tailed distributions.

Table 2. Descriptive statistics
The table presents descriptive statistics for the log returns of daily closing prices. Panel A reports statistics for the full sample periods (including both the estimation and testing windows), while Panel B covers only the testing windows. Obs denotes the number of return observations. Mean, Std, Skew, Kurt, Min, and Max refer to the sample mean, standard deviation, skewness, excess kurtosis, minimum, and maximum, respectively.

Panel A: Full sample periods							
	Obs	Mean	Std	Skew	Kurt	Min	Max
BTC	3,320	0.16	4.20	−0.51	10.81	−46.47	35.75
ETH	2,489	0.26	6.53	−3.18	68.63	−130.21	41.24
XRP	3,222	0.13	7.11	1.57	26.70	−61.64	102.75
DOGE	3,089	0.18	8.12	3.75	62.13	−58.10	151.62
LTC	3,320	0.08	6.23	1.14	23.28	−51.39	82.90
Panel B: Testing windows							
	Obs	Mean	Std	Skew	Kurt	Min	Max
BTC	2,820	0.15	3.90	−0.76	10.98	−46.47	22.51
ETH	1,989	0.28	5.53	−0.43	8.57	−55.07	29.01
XRP	2,722	0.10	6.64	2.01	35.06	−61.64	102.75
DOGE	2,589	0.26	7.43	4.20	77.83	−51.49	151.62
LTC	2,820	0.09	5.65	0.09	12.79	−51.39	51.03

3 METHODOLOGY

3.1 VaR calculation process

We employ a rolling estimation window (W_E) of length m to generate one-day-ahead volatility forecasts throughout a testing window (W_T) of length n. This procedure yields n volatility forecasts, $\widehat{\sigma}_{m+1}, \widehat{\sigma}_{m+2}, ..., \widehat{\sigma}_{m+n}$, where $\widehat{\sigma}_{m+1}$ is the forecasted volatility for the first day of W_T, and each subsequent $\widehat{\sigma}_t$ corresponds to the forecast for day $t-m$ within W_T. These one-day-ahead volatility forecasts are then used to compute n daily VaR forecasts at a given confidence level α, using the following formula:

$$VaR_{\alpha,t} = -\widehat{\sigma}_t \times F^{-1}_{1-\alpha} \times V_{t-1},\ t = 1, 2, ..., n, \tag{1}$$

where $VaR_{\alpha,t}$ represents the VaR forecast at confidence level α for day t, $\widehat{\sigma}_t$ is the one-day-ahead volatility forecast for day t, $F^{-1}_{1-\alpha}$ is the $(1-\alpha)$ quantile of the assumed return distribution, and V_{t-1} is the portfolio value on day $t-1$.

3.2 Risk forecast models

The one-day-ahead volatility forecasts, and consequently the corresponding VaR forecasts, are generated using five risk models. These include the four most commonly used models in practice (Danielsson et al., 2016) – historical simulation (HS), EWMA[4], normal GARCH (NGARCH), and Student's t GARCH (StGARCH) – as well as the t-StAR model[5] (Michaelides and Poudyal, 2024). The conditional variances for these models, except for HS, which is non-parametric, are provided in Appendix A.[6]

3.3 Backtesting and violation ratios

To assess and compare the forecasting performance of the various risk models, we implement a backtesting procedure as outlined in Danielsson (2011, Chapter 8). Specifically, we construct a sequence of violations, denoted by $\eta := (\eta_1, \eta_2, ..., \eta_n)$, where $\eta_t = 1$ if the ex-post realized return, r_t, exceeds the ex-ante VaR forecast, $VaR_{\alpha,t}$, on day t, and $\eta_t = 0$ otherwise. This sequence is then used to compute the VaR violation ratio as follows:

$$VR = \frac{\sum \eta}{(1-\alpha) \times n}. \tag{2}$$

This ratio's numerator represents the observed number of violations within the testing window, while its denominator corresponds to the expected number of violations based on the assumed confidence level. Among competing risk forecast models, violation ratios closest to 1 indicate the most accurate forecasts, while ratios below or

[4] The EWMA decay factor (λ) is set to 0.94, following the Riskmetrics framework of J.P. Morgan.

[5] The t-StAR model is estimated using the StAR function from the StReg package in R. Estimation begins with a random vector drawn from a uniform distribution, unless specific initial values are provided. To improve computational efficiency, the default initial values were used for the first estimation, with the final values from each run carried forward as the initial values for the next. Optimization was performed using the default BFGS algorithm, with no constraints imposed other than ensuring a positive-definite variance-covariance matrix via Cholesky decomposition. Prior to full-scale optimization, a sensitivity analysis was conducted to confirm that the estimation results were not significantly affected by the choice of initial values.

[6] Detailed discussions of the four standard models and the t-StAR model can be found in Danielsson (2011) and Michaelides and Poudyal (2024), respectively.

above 1 suggest over-forecasting or under-forecasting, respectively.

3.4 Coverage tests

To assess the adequacy of the risk models, we apply two standard coverage tests: the Kupiec (1995) unconditional coverage test and the Christoffersen (1998) conditional coverage test. The Kupiec test evaluates whether the observed proportion of violations matches the expected proportion. A statistically significant result indicates that a model fails to generate the correct number of violations, implying poor forecasting accuracy. The Christoffersen test extends the Kupiec test by evaluating not only the frequency of violations but also whether they occur independently over time. A statistically significant result suggests that a model fails in terms of either violation frequency or independence, or both. Together, these tests provide a more comprehensive assessment of risk model performance, complementing the violation ratio analysis.

4 EMPIRICAL RESULTS

Table 3 presents the VaR violation ratios for the risk forecast models evaluated in this paper. These ratios are reported for four confidence levels (α): the regulatory 99% level used by financial institutions under the Basel Accords (Panel A); the widely adopted 95% level (Panel B); the lower 90% level, often employed in risk management on the trading floor (Panel C); and the higher 99.9% level, typically used in applications such as economic capital, survival analysis, or long-term risk analysis for pension plans (Panel D); see Danielsson (2011, Chapter 4). For each confidence level, violation ratios are shown for three different estimation windows (W_E) of lengths 100, 250, and 500 days.[7] For models incorporating lag structures (i.e., NGARCH,

[7] Longer estimation windows (e.g., 1000 or 2000 days) were not employed due to data constraints, as their use would have significantly reduced the length of the testing windows.

StGARCH, and t-StAR), we report the best violation ratios achieved across estimations with varying lag orders[8]; the detailed results are provided in Appendix B. For each combination of cryptocurrency, estimation window, and confidence level, the risk forecast model(s) with violation ratios closest to 1 – indicating the best performance – are highlighted in bold.

Tables 4 and 5 complement the VaR violation ratios by presenting results from the Kupiec and Christoffersen coverage tests, respectively. These tables follow the same structure as Table 3, with each reported value representing a likelihood ratio (LR) test statistic and the corresponding p-value shown in parentheses. Statistical significance at the 5%, 1%, and 0.1% levels is denoted by *, **, and ***, respectively.[9]

The t-StAR model consistently delivers the most accurate and reliable VaR forecasts, attaining the best violation ratios in 49 out of 60 cases across all cryptocurrencies, estimation windows, and confidence levels. Its superior performance is further confirmed by coverage test results, as the model passes the Kupiec test in all 60 cases and the Christoffersen test in 57, demonstrating both the correct frequency and independence of violations.

HS ranks second based on violation ratios, outperforming other models in 10 out of 60 cases – approximately one-fifth as often as the t-StAR model. It generally produces violation ratios close to 1 and consistently passes the Kupiec test, indicating accurate violation frequency. However, its performance is less reliable in the Christoffersen test, especially at lower confidence levels (90% and 95%), suggesting the presence of time dependence in violations. Despite this, HS performs surprisingly well overall, outperforming all models except t-StAR. A notable limitation is

[8] Singularities are occasionally encountered when estimating volatility using GARCH-type models. In Appendix B (Tables B1 and B2), the number of such occurrences is shown in parentheses. These instances were excluded from the calculation of violation ratios. However, the number of singularities was small and had no material impact on the overall results. Importantly, the violation ratios reported in Table 3 are based exclusively on model specifications with lag orders that did not exhibit any singularities.

[9] Coverage tests cannot be computed when the observed number of violations is zero (i.e., when the violation ratio is exactly zero), as the corresponding test statistics are undefined in these cases. In Tables 4 and 5, such instances are represented by dashes (–).

its need for large estimation windows at higher confidence levels – for instance, at least 1,000 observations for the 99.9% level – which may be impractical for newer cryptocurrencies with limited historical data.

The EWMA and GARCH-type models (NGARCH and StGARCH) generally demonstrate weaker performance. Both EWMA and NGARCH tend to underestimate risk at higher confidence levels (99% and 99.9%) while overestimating it at the 90% level. Their forecast accuracy improves at the 95% level, where violation ratios are closer to 1 and coverage test results are more favorable. Although these models rarely outperform t-StAR or HS, they may still be appropriate for use at moderate confidence levels. As of StGARCH, it typically overestimates risk across most confidence levels. However, at the 99.9% level with a 100-day estimation window, violation ratios approach 1 and both coverage tests are successfully passed. This indicates that StGARCH may be better suited for applications requiring extremely high confidence levels, especially when using shorter estimation windows.

Table 3. VaR violation ratios
The table reports the VaR violation ratios. Each panel corresponds to one of four confidence levels (99%, 95%, 90%, and 99.9%), with results based on estimation windows (W_E) of 100, 250, and 500 days. Columns represent five models: historical simulation (HS), exponentially weighted moving average (EWMA), normal generalized autoregressive conditional heteroskedasticity (NGARCH), Student's t GARCH (StGARCH), and time-heterogeneous Student's t autoregressive (t-StAR). A VaR violation ratio measures the proportion of instances in which the actual loss exceeds the forecasted VaR. Bold values highlight the best-performing model(s) for each combination of cryptocurrency, estimation window, and confidence level – those with violation ratios closest to 1. Violation ratios below or above 1 suggest over-forecasting or under-forecasting, respectively.

Cryptocurrency	W_T	W_E	HS	EWMA	NGARCH	StGARCH	t-StAR
Panel A: $\alpha = 99\%$							
		100	**1.028**	2.234	2.199	0.745	0.887
BTC	2,820	250	0.745	2.234	2.057	0.426	**1.028**
		500	**1.028**	2.234	1.844	0.213	0.957
		100	1.307	2.313	1.860	0.503	**1.106**
ETH	1,989	250	0.754	2.313	1.911	0.452	**1.056**
		500	0.855	2.313	1.709	0.251	**1.006**
		100	**0.955**	1.653	1.947	0.588	1.065
XRP	2,722	250	0.918	1.653	1.653	0.147	**1.065**
		500	0.955	1.653	1.470	0.037	**0.992**
		100	1.120	1.275	1.390	0.502	**1.004**
DOGE	2,589	250	1.043	1.275	1.043	0.154	**1.004**
		500	1.236	1.275	**1.004**	0.116	**1.004**
		100	1.099	2.128	2.057	0.603	**0.957**
LTC	2,820	250	**0.922**	2.128	1.667	0.426	0.887
		500	**0.993**	2.128	1.489	0.142	1.064
Panel B: $\alpha = 95\%$							
		100	1.071	**0.979**	0.972	0.383	1.028
BTC	2,820	250	1.050	**0.979**	0.865	0.298	**1.021**
		500	1.057	**0.979**	0.887	0.248	1.050
		100	1.096	1.016	1.046	0.533	**0.985**
ETH	1,989	250	**1.006**	1.016	0.985	0.382	**1.006**
		500	0.955	**1.016**	0.925	0.302	0.975
		100	**0.985**	0.860	**0.985**	0.419	**0.985**
XRP	2,722	250	0.970	0.860	0.889	0.235	**1.007**
		500	**0.992**	0.860	0.808	0.162	1.043
		100	1.043	0.811	0.734	0.348	**1.004**
DOGE	2,589	250	0.896	0.811	0.703	0.185	**0.958**
		500	0.981	0.811	0.610	0.154	**0.997**
		100	1.021	0.887	0.908	0.433	**1.000**
LTC	2,820	250	0.950	0.887	0.766	0.277	**0.986**
		500	**1.007**	0.887	0.794	0.248	**0.993**
Panel C: $\alpha = 90\%$							
		100	0.986	0.706	0.734	0.429	**1.011**
BTC	2,820	250	1.028	0.706	0.727	0.337	**0.996**
		500	0.972	0.706	0.677	0.255	**0.989**
		100	1.026	0.734	0.799	0.503	**0.980**
ETH	1,989	250	1.051	0.734	0.744	0.442	**0.990**
		500	1.076	0.734	0.734	0.397	**0.990**
		100	0.974	0.683	0.746	0.437	**1.007**
XRP	2,722	250	**0.992**	0.683	0.702	0.320	1.025
		500	1.018	0.683	0.691	0.202	**1.014**
		100	1.004	0.668	0.606	0.351	**0.997**
DOGE	2,589	250	0.962	0.668	0.637	0.274	**0.985**
		500	0.977	0.668	0.552	0.201	**0.985**
		100	0.965	0.691	0.723	0.408	**1.007**
LTC	2,820	250	1.014	0.691	0.635	0.319	**1.004**
		500	1.028	0.691	0.638	0.270	**1.007**
Panel D: $\alpha = 99.9\%$							
		100	–	12.766	12.766	**1.064**	**1.064**
BTC	2,820	250	–	12.766	11.348	0.709	**1.064**
		500	–	12.766	7.801	0.355	**1.064**
		100	–	9.050	9.553	**1.006**	**1.006**
ETH	1,989	250	–	9.050	8.547	**1.006**	**1.006**
		500	–	9.050	7.039	0.000	**1.006**
		100	–	8.817	10.287	0.367	**1.102**
XRP	2,722	250	–	8.817	10.287	0.367	**1.102**
		500	–	8.817	6.980	0.000	**1.102**
		100	–	5.021	5.407	**1.159**	0.772
DOGE	2,589	250	–	5.021	5.021	0.386	**1.159**
		500	–	5.021	3.476	0.000	**0.772**
		100	–	10.638	10.993	**1.064**	**1.064**
LTC	2,820	250	–	10.638	9.574	0.355	**1.418**
		500	–	10.638	8.865	0.000	**1.064**

Table 4. Kupiec unconditional coverage tests
The table reports the results of Kupiec's (1995) unconditional coverage test. Each panel corresponds to one of four confidence levels (99%, 95%, 90%, and 99.9%), with results based on estimation windows (W_E) of 100, 250, and 500 days. Columns represent five models: historical simulation (HS), exponentially weighted moving average (EWMA), normal generalized autoregressive conditional heteroskedasticity (NGARCH), Student's t GARCH (StGARCH), and time-heterogeneous Student's t autoregressive (t-StAR). The Kupiec test evaluates whether the observed proportion of violations matches the expected proportion. A statistically significant result indicates that a model fails to generate the correct number of violations. The table reports the likelihood ratio (LR) test statistic and corresponding p-value (in parentheses), with significance denoted by * (5%), ** (1%), and *** (0.1%).

Cryptocurrency	W_T	W_E	HS	EWMA	NGARCH	StGARCH	t-StAR
Panel A: $\alpha = 99\%$							
		100	0.023 (0.880)	32.116 (0.000)***	30.500 (0.000)***	2.037 (0.154)	0.381 (0.537)
BTC	2,820	250	2.037 (0.154)	32.116 (0.000)***	24.369 (0.000)***	11.988 (0.001)**	0.023 (0.880)
		500	0.023 (0.880)	32.116 (0.000)***	16.243 (0.000)***	26.005 (0.000)***	0.052 (0.819)
		100	1.729 (0.189)	25.263 (0.000)***	11.861 (0.001)**	6.077 (0.014)*	0.219 (0.640)
ETH	1,989	250	1.327 (0.249)	25.263 (0.000)***	13.147 (0.000)***	7.566 (0.006)**	0.061 (0.804)
		500	0.446 (0.504)	25.263 (0.000)***	8.339 (0.004)**	16.085 (0.000)***	0.001 (0.980)
		100	0.056 (0.813)	9.802 (0.002)**	19.319 (0.000)***	5.483 (0.019)*	0.115 (0.734)
XRP	2,722	250	0.188 (0.665)	9.802 (0.002)**	9.802 (0.002)**	31.298 (0.000)***	0.115 (0.734)
		500	0.056 (0.813)	9.802 (0.002)**	5.295 (0.021)*	46.086 (0.000)***	0.002 (0.966)
		100	0.363 (0.547)	1.815 (0.178)	3.556 (0.059)	7.933 (0.005)**	0.000 (0.983)
DOGE	2,589	250	0.047 (0.828)	1.815 (0.178)	0.047 (0.828)	29.026 (0.000)***	0.000 (0.983)
		500	1.355 (0.244)	1.815 (0.178)	0.000 (0.983)	33.052 (0.000)***	0.000 (0.983)
		100	0.272 (0.602)	27.366 (0.000)***	24.369 (0.000)***	5.237 (0.022)*	0.052 (0.819)
LTC	2,820	250	0.178 (0.673)	27.366 (0.000)***	10.544 (0.001)**	11.988 (0.001)**	0.381 (0.537)
		500	0.001 (0.970)	27.366 (0.000)***	5.930 (0.015)*	32.985 (0.000)***	0.114 (0.736)
Panel B: $\alpha = 95\%$							
		100	0.730 (0.393)	0.068 (0.795)	0.121 (0.728)	73.139 (0.000)***	0.118 (0.731)
BTC	2,820	250	0.360 (0.548)	0.068 (0.795)	2.818 (0.093)	99.883 (0.000)***	0.067 (0.796)
		500	0.469 (0.493)	0.068 (0.795)	1.984 (0.159)	118.601 (0.000)***	0.360 (0.548)
		100	0.937 (0.333)	0.025 (0.874)	0.216 (0.642)	27.320 (0.000)***	0.022 (0.881)
ETH	1,989	250	0.003 (0.955)	0.025 (0.874)	0.022 (0.881)	51.760 (0.000)***	0.003 (0.955)
		500	0.213 (0.645)	0.025 (0.874)	0.602 (0.438)	69.514 (0.000)***	0.064 (0.800)
		100	0.034 (0.853)	2.956 (0.086)	0.034 (0.853)	61.377 (0.000)***	0.034 (0.853)
XRP	2,722	250	0.131 (0.717)	2.956 (0.086)	1.829 (0.176)	119.686 (0.000)***	0.006 (0.937)
		500	0.009 (0.923)	2.956 (0.086)	5.622 (0.018)*	152.979 (0.000)***	0.266 (0.606)
		100	0.247 (0.619)	5.182 (0.023)*	10.591 (0.001)**	76.670 (0.000)***	0.002 (0.960)
DOGE	2,589	250	1.522 (0.217)	5.182 (0.023)*	13.355 (0.000)***	134.466 (0.000)***	0.245 (0.621)
		500	0.049 (0.825)	5.182 (0.023)*	23.900 (0.000)***	148.997 (0.000)***	0.002 (0.968)
		100	0.067 (0.796)	1.984 (0.159)	1.300 (0.254)	60.143 (0.000)***	0.000 (1.000)
LTC	2,820	250	0.372 (0.542)	1.984 (0.159)	8.813 (0.003)**	107.590 (0.000)***	0.030 (0.862)
		500	0.007 (0.931)	1.984 (0.159)	6.734 (0.009)**	118.601 (0.000)***	0.007 (0.931)
Panel C: $\alpha = 90\%$							
		100	0.063 (0.801)	29.942 (0.000)***	28.442 (0.000)***	127.244 (0.000)***	0.035 (0.851)
BTC	2,820	250	0.250 (0.617)	29.942 (0.000)***	29.942 (0.000)***	180.726 (0.000)***	0.004 (0.950)
		500	0.254 (0.614)	29.942 (0.000)***	36.384 (0.000)***	240.321 (0.000)***	0.036 (0.850)
		100	0.144 (0.704)	17.063 (0.000)***	9.483 (0.002)**	65.640 (0.000)***	0.085 (0.770)
ETH	1,989	250	0.561 (0.454)	17.063 (0.000)***	15.739 (0.000)***	85.011 (0.000)***	0.020 (0.887)
		500	1.246 (0.264)	17.063 (0.000)***	17.063 (0.000)***	101.767 (0.000)***	0.020 (0.887)
		100	0.213 (0.644)	33.744 (0.000)***	21.244 (0.000)***	118.862 (0.000)***	0.013 (0.909)
XRP	2,722	250	0.020 (0.888)	33.744 (0.000)***	29.733 (0.000)***	185.591 (0.000)***	0.187 (0.665)
		500	0.094 (0.760)	33.744 (0.000)***	32.106 (0.000)***	277.199 (0.000)***	0.059 (0.809)
		100	0.005 (0.943)	35.439 (0.000)***	51.131 (0.000)***	157.322 (0.000)***	0.003 (0.953)
DOGE	2,589	250	0.425 (0.514)	35.439 (0.000)***	42.870 (0.000)***	206.846 (0.000)***	0.066 (0.798)
		500	0.150 (0.698)	35.439 (0.000)***	67.703 (0.000)***	264.710 (0.000)***	0.066 (0.798)
		100	0.322 (0.570)	33.075 (0.000)***	27.680 (0.000)***	138.451 (0.000)***	0.016 (0.900)
LTC	2,820	250	0.063 (0.802)	33.075 (0.000)***	47.406 (0.000)***	192.594 (0.000)***	0.004 (0.950)
		500	0.250 (0.617)	33.075 (0.000)***	46.423 (0.000)***	228.987 (0.000)***	0.016 (0.900)
Panel D: $\alpha = 99.9\%$							
		100	–	117.401 (0.000)***	117.401 (0.000)***	0.011 (0.915)	0.011 (0.915)
BTC	2,820	250	–	117.401 (0.000)***	97.399 (0.000)***	0.266 (0.606)	0.011 (0.915)
		500	–	117.401 (0.000)***	52.160 (0.000)***	1.568 (0.211)	0.011 (0.915)
		100	–	47.406 (0.000)***	51.883 (0.000)***	0.000 (0.994)	0.000 (0.994)
ETH	1,989	250	–	47.406 (0.000)***	43.041 (0.000)***	0.000 (0.994)	0.000 (0.994)
		500	–	47.406 (0.000)***	30.691 (0.000)***	–	0.000 (0.994)
		100	–	62.092 (0.000)***	80.207 (0.000)***	1.442 (0.230)	0.028 (0.868)
XRP	2,722	250	–	62.092 (0.000)***	80.207 (0.000)***	1.442 (0.230)	0.028 (0.868)
		500	–	62.092 (0.000)***	41.378 (0.000)***	–	0.028 (0.868)
		100	–	21.176 (0.000)***	24.486 (0.000)***	0.062 (0.803)	0.146 (0.703)
DOGE	2,589	250	–	21.176 (0.000)***	21.176 (0.000)***	1.276 (0.259)	0.062 (0.803)
		500	–	21.176 (0.000)***	9.621 (0.002)**	–	0.146 (0.703)
		100	–	87.771 (0.000)***	92.552 (0.000)***	0.011 (0.915)	0.011 (0.915)
LTC	2,820	250	–	87.771 (0.000)***	73.840 (0.000)***	1.568 (0.211)	0.437 (0.509)
		500	–	87.771 (0.000)***	64.922 (0.000)***	–	0.011 (0.915)

Table 5. Christoffersen conditional coverage tests

The table reports the results of Christoffersen's (1998) conditional coverage test. Each panel corresponds to one of four confidence levels (99%, 95%, 90%, and 99.9%), with results based on estimation windows (W_E) of 100, 250, and 500 days. Columns represent five models: historical simulation (HS), exponentially weighted moving average (EWMA), normal generalized autoregressive conditional heteroskedasticity (NGARCH), Student's t GARCH (StGARCH), and time-heterogeneous Student's t autoregressive (t-StAR). The Christoffersen test jointly evaluates the frequency of violations and their independence over time. A statistically significant result indicates that a model fails in one or both of these aspects. The table reports the likelihood ratio (LR) test statistic and corresponding p-value (in parentheses), with significance denoted by * (5%), ** (1%), and *** (0.1%).

Cryptocurrency	W_T	W_E	HS	EWMA	NGARCH	StGARCH	t-StAR
Panel A: $\alpha = 99\%$							
BTC	2,820	100	4.437 (0.109)	38.037 (0.000)***	30.772 (0.000)***	4.130 (0.127)	0.829 (0.661)
		250	8.880 (0.012)*	38.037 (0.000)***	24.404 (0.000)***	12.090 (0.002)**	0.626 (0.731)
		500	4.437 (0.109)	38.037 (0.000)***	16.245 (0.000)***	26.031 (0.000)***	0.575 (0.750)
ETH	1,989	100	2.601 (0.272)	25.954 (0.000)***	11.990 (0.002)**	6.178 (0.046)*	0.711 (0.701)
		250	1.555 (0.460)	25.954 (0.000)***	14.740 (0.001)**	7.648 (0.022)*	0.510 (0.775)
		500	0.739 (0.691)	25.954 (0.000)***	9.522 (0.009)**	16.110 (0.000)***	0.407 (0.816)
XRP	2,722	100	1.383 (0.501)	13.890 (0.001)**	19.321 (0.000)***	8.504 (0.014)*	1.115 (0.573)
		250	1.638 (0.441)	13.890 (0.001)**	9.884 (0.007)**	31.310 (0.000)***	0.740 (0.691)
		500	1.383 (0.501)	13.890 (0.001)**	6.489 (0.039)*	46.087 (0.000)***	0.543 (0.762)
DOGE	2,589	100	4.483 (0.106)	2.667 (0.264)	4.571 (0.102)	8.064 (0.018)*	0.528 (0.768)
		250	1.184 (0.553)	2.667 (0.264)	0.617 (0.735)	29.038 (0.000)***	0.528 (0.768)
		500	4.794 (0.091)	2.667 (0.264)	0.528 (0.768)	33.059 (0.000)***	0.528 (0.768)
LTC	2,820	100	8.480 (0.014)*	29.148 (0.000)***	24.845 (0.000)***	5.443 (0.066)	0.575 (0.750)
		250	1.559 (0.459)	29.148 (0.000)***	11.925 (0.003)**	12.090 (0.002)**	0.829 (0.661)
		500	4.669 (0.097)	29.148 (0.000)***	7.200 (0.027)*	32.996 (0.000)***	0.759 (0.684)
Panel B: $\alpha = 95\%$							
BTC	2,820	100	7.654 (0.022)*	3.803 (0.149)	1.749 (0.417)	75.746 (0.000)***	3.946 (0.139)
		250	6.413 (0.041)*	3.803 (0.149)	8.108 (0.017)*	101.153 (0.000)***	3.679 (0.159)
		500	9.621 (0.008)**	3.803 (0.149)	4.001 (0.135)	119.481 (0.000)***	1.020 (0.601)
ETH	1,989	100	1.639 (0.441)	1.571 (0.456)	0.257 (0.880)	27.461 (0.000)***	0.189 (0.910)
		250	1.680 (0.432)	1.571 (0.456)	0.981 (0.612)	51.856 (0.000)***	0.200 (0.905)
		500	0.687 (0.709)	1.571 (0.456)	2.213 (0.331)	70.434 (0.000)***	0.080 (0.961)
XRP	2,722	100	11.032 (0.004)**	5.778 (0.056)	0.344 (0.842)	61.411 (0.000)***	4.060 (0.131)
		250	16.410 (0.000)***	5.778 (0.056)	4.084 (0.130)	120.418 (0.000)***	8.165 (0.017)*
		500	20.093 (0.000)***	5.778 (0.056)	5.694 (0.058)	153.337 (0.000)***	4.184 (0.123)
DOGE	2,589	100	9.912 (0.007)**	5.630 (0.060)	13.204 (0.001)**	78.263 (0.000)***	0.347 (0.841)
		250	13.125 (0.001)**	5.630 (0.060)	15.539 (0.000)***	134.916 (0.000)***	2.830 (0.243)
		500	14.661 (0.001)**	5.630 (0.060)	28.876 (0.000)***	149.309 (0.000)***	0.034 (0.983)
LTC	2,820	100	8.792 (0.012)*	4.001 (0.135)	4.075 (0.130)	60.461 (0.000)***	0.553 (0.759)
		250	10.030 (0.007)**	4.001 (0.135)	13.548 (0.001)**	110.023 (0.000)***	1.448 (0.485)
		500	3.093 (0.213)	4.001 (0.135)	10.711 (0.005)**	119.481 (0.000)***	0.175 (0.916)
Panel C: $\alpha = 90\%$							
BTC	2,820	100	7.477 (0.024)*	39.688 (0.000)***	30.797 (0.000)***	132.746 (0.000)***	2.726 (0.256)
		250	5.016 (0.081)	39.688 (0.000)***	31.790 (0.000)***	180.925 (0.000)***	1.504 (0.471)
		500	9.838 (0.007)**	39.688 (0.000)***	39.292 (0.000)***	240.801 (0.000)***	6.217 (0.045)*
ETH	1,989	100	3.678 (0.159)	20.793 (0.000)***	11.051 (0.004)**	67.316 (0.000)***	2.146 (0.342)
		250	1.443 (0.486)	20.793 (0.000)***	16.136 (0.000)***	85.328 (0.000)***	1.217 (0.544)
		500	3.071 (0.215)	20.793 (0.000)***	17.071 (0.000)***	103.880 (0.000)***	0.035 (0.983)
XRP	2,722	100	12.399 (0.002)**	39.082 (0.000)***	23.646 (0.000)***	120.053 (0.000)***	9.430 (0.009)**
		250	9.415 (0.009)**	39.082 (0.000)***	30.772 (0.000)***	185.609 (0.000)***	1.868 (0.393)
		500	21.099 (0.000)***	39.082 (0.000)***	32.474 (0.000)***	277.802 (0.000)***	3.385 (0.184)
DOGE	2,589	100	8.112 (0.017)*	35.457 (0.000)***	52.807 (0.000)***	157.875 (0.000)***	0.081 (0.960)
		250	19.348 (0.000)***	35.457 (0.000)***	42.900 (0.000)***	210.853 (0.000)***	3.617 (0.164)
		500	15.833 (0.000)***	35.457 (0.000)***	70.290 (0.000)***	266.843 (0.000)***	2.885 (0.236)
LTC	2,820	100	2.798 (0.247)	38.417 (0.000)***	31.686 (0.000)***	140.578 (0.000)***	0.098 (0.952)
		250	7.559 (0.023)*	38.417 (0.000)***	50.222 (0.000)***	195.407 (0.000)***	1.789 (0.409)
		500	7.770 (0.021)*	38.417 (0.000)***	46.641 (0.000)***	230.542 (0.000)***	1.687 (0.430)
Panel D: $\alpha = 99.9\%$							
BTC	2,820	100	–	117.891 (0.000)***	120.337 (0.000)***	0.018 (0.991)	0.018 (0.991)
		250	–	117.891 (0.000)***	98.177 (0.000)***	0.269 (0.874)	0.018 (0.991)
		500	–	117.891 (0.000)***	54.092 (0.000)***	1.568 (0.456)	0.018 (0.991)
ETH	1,989	100	–	47.735 (0.000)***	52.249 (0.000)***	0.004 (0.998)	0.004 (0.998)
		250	–	47.735 (0.000)***	43.335 (0.000)***	0.004 (0.998)	0.004 (0.998)
		500	–	47.735 (0.000)***	30.889 (0.000)***	–	0.004 (0.998)
XRP	2,722	100	–	62.519 (0.000)***	80.789 (0.000)***	1.443 (0.486)	0.034 (0.983)
		250	–	62.519 (0.000)***	80.789 (0.000)***	1.443 (0.486)	0.034 (0.983)
		500	–	62.519 (0.000)***	41.646 (0.000)***	–	0.034 (0.983)
DOGE	2,589	100	–	21.307 (0.000)***	24.639 (0.000)***	0.069 (0.966)	0.149 (0.928)
		250	–	21.307 (0.000)***	21.307 (0.000)***	1.277 (0.528)	0.069 (0.966)
		500	–	21.307 (0.000)***	9.684 (0.008)**	–	0.149 (0.928)
LTC	2,820	100	–	88.416 (0.000)***	93.242 (0.000)***	0.018 (0.991)	0.018 (0.991)
		250	–	88.416 (0.000)***	74.362 (0.000)***	1.568 (0.456)	0.448 (0.799)
		500	–	88.416 (0.000)***	65.369 (0.000)***	–	0.018 (0.991)

5 CONCLUDING REMARKS

This paper evaluates the effectiveness of the t-StAR model, proposed by Michaelides and Poudyal (2024), in forecasting VaR for cryptocurrencies. Benchmarked against standard risk forecast models commonly used in practice – namely, HS, EWMA, NGARCH, and StGARCH – the t-StAR model demonstrates superior performance across various cryptocurrencies, estimation windows, and confidence levels. Its consistent accuracy underscores both its robustness and potential for application in highly volatile settings.

The findings offer actionable insights for practitioners managing the market risk of volatile portfolios – such as those containing cryptocurrencies – as well as for regulators concerned with financial stability and standard setters involved in shaping risk management practices. First, the t-StAR model stands out as the most reliable option, consistently achieving violation ratios close to 1 and passing both the Kupiec and Christoffersen coverage tests. This makes it suitable for a wide range of applications, from routine internal risk monitoring to formal regulatory reporting under Basel capital adequacy requirements. Nevertheless, its computational demands may limit its practicality in time-sensitive or resource-constrained settings, unless appropriate infrastructure or technical expertise is available.

Second, for those who prefer simpler methods, HS remains a viable choice due to its transparency and ease of implementation. However, it often exhibits time-dependent violations, indicating a limited ability to adapt to changing volatility. Additionally, at higher confidence levels, HS requires long estimation windows, which may necessitate data that is not always readily available. Third, EWMA and NGARCH may perform reasonably well at moderate confidence levels (e.g., 95%) but tend to be less reliable overall and should be used with caution. Fourth, while StGARCH generally overestimates risk, it shows promise at extreme confidence levels (e.g., 99.9%) when used with short estimation windows (e.g., 100 days), suggesting potential for specific applications. Ultimately, model selection should be guided by the required confidence level, data availability, and the trade-off between complexity, speed, and accuracy.

Although this study benchmarks the t-StAR model against standard approaches, there is growing interest in hybrid models (e.g., Kuester et al., 2006) and machine learning techniques (e.g., Arian et al., 2022) for VaR forecasting. These emerging methods may offer greater flexibility and better capture complex dynamics in financial time series. Nonetheless, they often lack interpretability, which limits their practical and regulatory adoption. Future research could explore how the t-StAR model compares with these novel approaches in terms of forecasting performance.

Finally, this study focuses on a limited subset of cryptocurrencies. With over 20,000 currently in existence, most of which are newly launched and lack sufficient historical data, reliable backtesting remains challenging. Future work could extend the evaluation of the t-StAR model to a broader range of cryptocurrencies, including highly volatile altcoins and tokens issued on existing blockchains.

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Appendix A: Risk Forecast Models

Historical Simulation (HS). A non-parametric method where the VaR at confidence level α is defined as the negative $(T \times (1-\alpha))^{th}$ value in the sorted return vector, multiplied be the monetary value of the portfolio.

Exponentially Weighted Moving Average (EWMA). The conditional variance of EWMA is:

$$\widehat{\sigma}_t^2 = (1-\lambda) r_{t-1}^2 + \lambda \widehat{\sigma}_{t-1}^2,$$

where $0 < \lambda < 1$ denotes a decay factor.

Normal GARCH (NGARCH). The conditional variance of NGARCH(p, q) is:

$$\widehat{\sigma}_t^2 = \omega + \textstyle\sum_{i=1}^{p} \alpha_i r_{t-i}^2 + \sum_{j=1}^{q} \beta_j \widehat{\sigma}_{t-j}^2,$$

where $\omega,\ \{\alpha_i,\ i = 1, 2, ..., p\}$, and $\{\beta_j,\ j = 1, 2, ..., q\}$ refer to estimated parameters. Restrictions imposed on parameters are necessary to ensure positive $\widehat{\sigma}_t^2$.

Student's *t* GARCH (StGARCH). The residuals of StGARCH$(p, q; \nu)$ are Student's t distributed with ν degrees of freedom. The degrees of freedom, ν, is estimated as an extra parameter along with the NGARCH model parameters. Similar to NGARCH, restrictions imposed on parameters are necessary.

Time-heterogeneous Student's *t* Autoregressive (t-StAR). The conditional variance of t-StAR$(p; \nu)$ is:

$$\widehat{\sigma}_t^2 = \left(\frac{\nu \omega^2}{\nu + p - 2} \right) \left[1 + \frac{1}{\nu} \left[\textstyle\sum_{i=1}^{p} \left(r_{t-i} - \mu_{t-i}(t) \right) \mathbf{\Sigma}^{-1} \left(r_{t-i} - \mu_{t-i}(t) \right) \right] \right],$$

where ω^2 is a scaling variance constant, $\mathbf{\Sigma}$ is the variance-covariance matrix, and $\mu_{t-i}(t)$ is the time-varying unconditional mean of r_{t-i}, for $i = 1, 2, ..., p$. The degrees of freedom of the Student's t distribution, ν, is an estimable parameter. The degrees of freedom for the conditional Student's t distribution is equal to ν plus the number of lagged conditioning variables, p.

Appendix B: Additional Tables

Table B1. VaR violation ratios for NGARCH(p, q)
The table reports the VaR violation ratios for NGARCH(p, q). Each panel corresponds to one of four confidence levels (99%, 95%, 90%, and 99.9%), with results based on estimation windows (W_E) of 100, 250, and 500 days. Columns represent NGARCH(p, q) specifications with lag orders p and q ranging from 1 to 3. A VaR violation ratio measures the proportion of instances in which the actual loss exceeds the forecasted VaR. Violation ratios closest to 1 indicate the specification with the best forecast accuracy, while ratios below or above 1 suggest over-forecasting or under-forecasting, respectively. Numbers in parentheses indicate the number of singularities encountered during the VaR calculation process. When singularities occur, they are excluded from the violation ratio calculation.

Cryptocurrency	W_E	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)	(3,1)	(3,2)	(3,3)
Panel A: $\alpha = 99\%$										
BTC	100	2.199	2.199	2.131 (4)	2.308 (4)	2.237 (4)	2.202 (4)	2.202 (4)	2.166 (4)	2.131 (4)
	250	2.128	2.057	2.005 (27)	2.077 (27)	1.969 (27)	1.969 (27)	2.112 (27)	2.077 (27)	2.005 (27)
	500	1.950	1.844	1.915	2.021	1.879	1.915	2.057	1.915	1.915
ETH	100	1.860	1.860	2.067 (5)	1.966 (5)	1.966 (5)	2.117 (5)	2.016 (5)	2.117 (5)	2.218 (5)
	250	2.011	1.911	1.821 (12)	1.922 (12)	1.922 (12)	1.821 (12)	2.023 (12)	2.023 (12)	1.922 (12)
	500	1.709	1.760	1.760	1.760	1.860	1.760	1.760	1.911	1.810
XRP	100	1.947	2.168	2.139 (10)	1.991 (10)	2.286 (10)	2.286 (10)	2.176 (10)	2.471 (10)	2.618 (10)
	250	1.690	1.653	1.766 (4)	1.729 (4)	1.766 (4)	1.840 (4)	1.950 (4)	1.913 (4)	2.060 (4)
	500	1.470	1.543	1.543	1.470	1.690	1.800	1.690	1.800	1.837
DOGE	100	1.390	1.699	1.669 (12)	1.591 (12)	1.746 (12)	1.863 (12)	1.940 (12)	1.979 (12)	1.979 (12)
	250	1.081	1.043	1.120	1.120	1.081	1.159	1.159	1.120	1.236
	500	1.004	1.120	1.004	1.004	1.197	1.159	1.043	1.159	1.120
LTC	100	2.057	2.092	2.172 (12)	2.208 (12)	2.066 (12)	2.138 (14)	2.281 (14)	2.352 (14)	2.459 (14)
	250	1.667	1.738	1.925 (15)	1.747 (15)	1.818 (15)	1.783 (15)	1.889 (15)	1.889 (15)	1.961 (15)
	500	1.525	1.560	1.667	1.489	1.525	1.525	1.525	1.489	1.525
Panel B: $\alpha = 95\%$										
BTC	100	0.936	0.972	1.001 (4)	0.959 (4)	1.016 (4)	1.044 (4)	1.009 (4)	1.009 (4)	1.087 (4)
	250	0.851	0.865	0.909 (27)	0.859 (27)	0.895 (27)	0.938 (27)	0.881 (27)	0.924 (27)	0.995 (27)
	500	0.837	0.830	0.858	0.823	0.887	0.879	0.837	0.865	0.872
ETH	100	1.046	1.106	1.089 (5)	1.099 (5)	1.119 (5)	1.129 (5)	1.069 (5)	1.149 (5)	1.159 (5)
	250	0.975	0.985	0.991 (12)	0.981 (12)	1.012 (12)	1.042 (12)	0.981 (12)	1.002 (12)	1.042 (12)
	500	0.804	0.845	0.915	0.825	0.915	0.915	0.855	0.905	0.925
XRP	100	0.940	0.985	1.032 (10)	0.973 (10)	1.018 (10)	1.069 (10)	1.032 (10)	1.069 (10)	1.143 (10)
	250	0.889	0.889	0.912 (4)	0.898 (4)	0.920 (4)	0.942 (4)	0.949 (4)	0.949 (4)	0.964 (4)
	500	0.705	0.720	0.757	0.720	0.757	0.808	0.749	0.786	0.808
DOGE	100	0.734	0.711	0.761 (12)	0.761 (12)	0.792 (12)	0.838 (12)	0.776 (12)	0.799 (12)	0.893 (12)
	250	0.664	0.672	0.672	0.672	0.680	0.703	0.688	0.680	0.703
	500	0.533	0.556	0.548	0.510	0.579	0.610	0.518	0.579	0.595
LTC	100	0.908	0.908	0.962 (12)	0.947 (12)	0.947 (12)	0.969 (14)	0.998 (14)	0.984 (14)	1.012 (14)
	250	0.766	0.752	0.813 (15)	0.813 (15)	0.791 (15)	0.848 (15)	0.820 (15)	0.799 (15)	0.841 (15)
	500	0.688	0.709	0.745	0.681	0.723	0.794	0.695	0.730	0.794
Panel C: $\alpha = 90\%$										
BTC	100	0.713	0.734	0.749 (4)	0.724 (4)	0.742 (4)	0.795 (4)	0.774 (4)	0.781 (4)	0.827 (4)
	250	0.706	0.727	0.748 (27)	0.705 (27)	0.752 (27)	0.766 (27)	0.745 (27)	0.755 (27)	0.773 (27)
	500	0.649	0.652	0.670	0.652	0.667	0.663	0.667	0.677	0.674
ETH	100	0.794	0.799	0.806 (5)	0.822 (5)	0.842 (5)	0.867 (5)	0.827 (5)	0.847 (5)	0.867 (5)
	250	0.739	0.744	0.749 (12)	0.718 (12)	0.738 (12)	0.764 (12)	0.718 (12)	0.744 (12)	0.769 (12)
	500	0.679	0.684	0.714	0.689	0.719	0.734	0.689	0.719	0.734
XRP	100	0.731	0.746	0.771 (10)	0.756 (10)	0.782 (10)	0.804 (10)	0.815 (10)	0.808 (10)	0.811 (10)
	250	0.702	0.702	0.717 (4)	0.710 (4)	0.754 (4)	0.747 (4)	0.747 (4)	0.780 (4)	0.776 (4)
	500	0.606	0.621	0.643	0.625	0.669	0.680	0.654	0.669	0.691
DOGE	100	0.603	0.606	0.660 (12)	0.652 (12)	0.644 (12)	0.702 (12)	0.675 (12)	0.660 (12)	0.726 (12)
	250	0.576	0.579	0.591	0.599	0.614	0.618	0.618	0.618	0.637
	500	0.494	0.518	0.525	0.494	0.518	0.548	0.502	0.518	0.552
LTC	100	0.716	0.723	0.744 (12)	0.759 (12)	0.755 (12)	0.763 (14)	0.759 (14)	0.763 (14)	0.784 (14)
	250	0.631	0.635	0.663 (15)	0.645 (15)	0.649 (15)	0.667 (15)	0.667 (15)	0.656 (15)	0.674 (15)
	500	0.592	0.599	0.621	0.599	0.599	0.617	0.596	0.603	0.638
Panel D: $\alpha = 99.9\%$										
BTC	100	12.766	13.121	13.494 (4)	12.074 (4)	12.784 (4)	13.849 (4)	12.074 (4)	13.139 (4)	13.849 (4)
	250	11.348	11.348	11.099 (27)	11.457 (27)	11.099 (27)	11.099 (27)	10.741 (27)	11.099 (27)	10.741 (27)
	500	8.156	8.511	8.865	7.801	8.156	8.511	7.801	8.156	8.865
ETH	100	9.553	10.055	10.585 (5)	10.081 (5)	10.585 (5)	12.097 (5)	9.073 (5)	10.585 (5)	12.601 (5)
	250	8.547	9.553	9.105 (12)	7.587 (12)	8.599 (12)	9.105 (12)	8.093 (12)	8.599 (12)	9.105 (12)
	500	7.541	7.541	7.541	7.039	7.039	8.044	7.039	7.039	8.044
XRP	100	10.287	11.389	11.799 (10)	11.431 (10)	12.537 (10)	12.168 (10)	11.799 (10)	11.799 (10)	12.168 (10)
	250	10.287	10.287	9.934 (4)	10.670 (4)	11.038 (4)	10.302 (4)	9.934 (4)	11.405 (4)	10.302 (4)
	500	6.980	7.348	8.082	7.348	7.348	8.082	8.450	8.450	8.817
DOGE	100	5.407	6.180	6.209 (12)	5.821 (12)	6.597 (12)	6.597 (12)	6.597 (12)	7.761 (12)	8.149 (12)
	250	5.021	5.407	5.407	5.407	5.794	5.794	5.407	5.407	5.407
	500	3.476	3.476	3.862	3.476	3.476	4.249	3.476	3.476	3.862
LTC	100	10.993	10.993	11.396 (12)	10.684 (12)	11.040 (12)	11.048 (14)	11.048 (14)	12.117 (14)	12.473 (14)
	250	9.574	9.574	9.269 (15)	9.626 (15)	9.269 (15)	9.626 (15)	8.913 (15)	8.556 (15)	9.269 (15)
	500	8.865	8.865	9.220	9.220	8.865	9.574	9.220	9.574	9.929

Table B2. VaR violation ratios for StGARCH$(p, q; \nu)$

The table reports the VaR violation ratios for StGARCH$(p, q; \nu)$ with ν degrees of freedom. Each panel corresponds to one of four confidence levels (99%, 95%, 90%, and 99.9%), with results based on estimation windows (W_E) of 100, 250, and 500 days. Columns represent StGARCH$(p, q; \nu)$ specifications with lag orders p and q ranging from 1 to 3. A VaR violation ratio measures the proportion of instances in which the actual loss exceeds the forecasted VaR. Violation ratios closest to 1 indicate the specification with the best forecast accuracy, while ratios below or above 1 suggest over-forecasting or under-forecasting, respectively. Numbers in parentheses indicate the number of singularities encountered during the VaR calculation process. When singularities occur, they are excluded from the violation ratio calculation.

Cryptocurrency	W_E	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)	(3,1)	(3,2)	(3,3)
Panel A: $\alpha = 99\%$										
BTC	100	0.674	0.745	0.780 (1)	0.674 (1)	0.674 (1)	0.639 (1)	0.780 (1)	0.674 (1)	0.709 (1)
	250	0.390	0.426	0.390 (1)	0.355 (1)	0.390 (1)	0.355 (1)	0.355 (1)	0.390 (1)	0.390 (1)
	500	0.213	0.213	0.213	0.177	0.213	0.213	0.213	0.213	0.213
ETH	100	0.452	0.503	0.503 (1)	0.604 (1)	0.654 (1)	0.654 (1)	0.805 (1)	0.755 (1)	0.805 (1)
	250	0.352	0.302	0.302	0.452	0.352	0.452	0.352	0.402	0.452
	500	0.251	0.251	0.251	0.251	0.251	0.251	0.201	0.251	0.251
XRP	100	0.404	0.478	0.331	0.478	0.588	0.514	0.514	0.588	0.514
	250	0.073	0.073	0.073	0.110	0.110	0.110	0.147	0.110	0.110
	500	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037
DOGE	100	0.309	0.386	0.425	0.348	0.425	0.502	0.386	0.502 (1)	0.580 (1)
	250	0.154	0.154	0.154	0.154	0.154	0.154	0.154	0.154	0.154
	500	0.116	0.116	0.116	0.116	0.116	0.116	0.077	0.116	0.116
LTC	100	0.496	0.496	0.603	0.496	0.496	0.603	0.461	0.496	0.567
	250	0.319	0.319	0.355	0.319	0.319	0.355	0.390	0.390	0.426
	500	0.071	0.071	0.071	0.071	0.142	0.142	0.106	0.142	0.142
Panel B: $\alpha = 95\%$										
BTC	100	0.383	0.383	0.426 (1)	0.419 (1)	0.419 (1)	0.454 (1)	0.440 (1)	0.404 (1)	0.468 (1)
	250	0.298	0.298	0.312 (1)	0.312 (1)	0.326 (1)	0.341 (1)	0.312 (1)	0.319 (1)	0.341 (1)
	500	0.213	0.227	0.234	0.220	0.227	0.248	0.234	0.234	0.241
ETH	100	0.533	0.513	0.563 (1)	0.523 (1)	0.533 (1)	0.584 (1)	0.553 (1)	0.553 (1)	0.604 (1)
	250	0.312	0.362	0.362	0.312	0.372	0.382	0.302	0.342	0.382
	500	0.261	0.271	0.271	0.251	0.292	0.302	0.251	0.282	0.292
XRP	100	0.345	0.360	0.360	0.360	0.389	0.404	0.404	0.411	0.419
	250	0.213	0.206	0.206	0.220	0.220	0.220	0.235	0.220	0.220
	500	0.154	0.154	0.162	0.147	0.147	0.162	0.140	0.147	0.162
DOGE	100	0.270	0.294	0.317	0.286	0.317	0.348	0.348	0.363 (1)	0.371 (1)
	250	0.162	0.185	0.178	0.170	0.178	0.170	0.147	0.170	0.178
	500	0.147	0.147	0.154	0.147	0.147	0.147	0.139	0.147	0.154
LTC	100	0.340	0.369	0.383	0.383	0.383	0.404	0.397	0.390	0.433
	250	0.220	0.220	0.248	0.234	0.255	0.277	0.262	0.277	0.277
	500	0.227	0.227	0.248	0.234	0.241	0.241	0.227	0.234	0.227
Panel C: $\alpha = 90\%$										
BTC	100	0.415	0.429	0.440 (1)	0.415 (1)	0.419 (1)	0.429 (1)	0.411 (1)	0.422 (1)	0.440 (1)
	250	0.333	0.337	0.348 (1)	0.333 (1)	0.333 (1)	0.348 (1)	0.333 (1)	0.337 (1)	0.344 (1)
	500	0.248	0.252	0.252	0.245	0.255	0.252	0.245	0.252	0.252
ETH	100	0.493	0.503	0.503 (1)	0.498 (1)	0.543 (1)	0.543 (1)	0.498 (1)	0.523 (1)	0.518 (1)
	250	0.427	0.427	0.427	0.422	0.432	0.442	0.412	0.437	0.442
	500	0.357	0.367	0.372	0.357	0.392	0.392	0.372	0.387	0.397
XRP	100	0.367	0.364	0.371	0.404	0.404	0.404	0.437	0.434	0.426
	250	0.253	0.276	0.287	0.268	0.298	0.316	0.283	0.298	0.320
	500	0.180	0.173	0.187	0.176	0.176	0.195	0.176	0.176	0.202
DOGE	100	0.305	0.317	0.317	0.317	0.336	0.328	0.351	0.359 (1)	0.340 (1)
	250	0.209	0.251	0.251	0.201	0.267	0.274	0.212	0.263	0.274
	500	0.166	0.189	0.201	0.174	0.197	0.201	0.185	0.189	0.197
LTC	100	0.344	0.351	0.369	0.376	0.362	0.376	0.379	0.365	0.408
	250	0.294	0.298	0.316	0.294	0.284	0.316	0.301	0.294	0.319
	500	0.238	0.248	0.266	0.234	0.245	0.255	0.241	0.245	0.270
Panel D: $\alpha = 99.9\%$										
BTC	100	1.064	1.064	1.064 (1)	1.064 (1)	1.774 (1)	1.419 (1)	1.419 (1)	1.774 (1)	1.419 (1)
	250	0.709	0.709	1.064 (1)	0.709 (1)	0.709 (1)	1.064 (1)	0.709 (1)	0.709 (1)	1.064 (1)
	500	0.355	0.355	0.355	0.355	0.355	0.355	0.355	0.355	0.355
ETH	100	1.006	1.006	1.006 (1)	1.509 (1)	2.012 (1)	2.012 (1)	1.509 (1)	2.012 (1)	2.012 (1)
	250	1.006	1.006	1.006	1.006	1.006	1.006	1.006	1.006	1.006
	500	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
XRP	100	0.367	0.367	0.367	0.367	0.367	0.367	0.000	0.367	0.367
	250	0.367	0.367	0.000	0.367	0.367	0.000	0.367	0.367	0.000
	500	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
DOGE	100	0.386	0.772	0.772	0.772	0.772	0.772	1.159	1.159 (1)	1.159 (1)
	250	0.386	0.386	0.386	0.386	0.386	0.386	0.386	0.386	0.386
	500	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LTC	100	0.709	0.709	1.418	0.709	0.709	1.418	0.709	0.709	1.064
	250	0.355	0.000	0.355	0.000	0.000	0.000	0.000	0.000	0.000
	500	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table B3. VaR violation ratios for t-StAR$(p;\nu)$

The table reports the VaR violation ratios for t-StAR$(p;\nu)$ with ν degrees of freedom. Each panel corresponds to one of four confidence levels (99%, 95%, 90%, and 99.9%), with results based on estimation windows (W_E) of 100, 250, and 500 days and lag order p ranging from 1 to 3. Columns represent t-StAR$(p;\nu)$ specifications with degrees of freedom ν ranging from 1 to 10. A VaR violation ratio measures the proportion of instances in which the actual loss exceeds the forecasted VaR. Violation ratios closest to 1 indicate the specification with the best forecast accuracy, while ratios below or above 1 suggest over-forecasting or under-forecasting, respectively.

Panel A: $\alpha = 99\%$												
Cryptocurrency	W_E	$p\backslash\nu$	1	2	3	4	5	6	7	8	9	10
		1	–	0.709	1.809	2.660	3.475	4.291	4.752	5.461	5.957	6.135
	100	2	0.319	0.816	1.277	1.915	2.482	3.121	3.440	3.830	4.149	4.397
		3	0.532	0.887	1.348	1.702	2.128	2.376	2.837	3.050	3.333	3.582
		1	–	0.390	1.277	2.057	3.085	3.865	4.716	5.142	5.603	5.957
BTC	250	2	0.284	0.709	1.241	1.667	2.270	2.766	3.298	3.652	4.113	4.255
		3	0.745	1.028	1.596	2.057	2.411	2.837	3.014	3.227	3.475	3.688
		1	–	0.461	1.348	2.340	3.156	3.723	4.220	4.716	5.071	5.532
	500	2	0.213	0.638	1.206	1.915	2.730	2.943	3.262	3.546	3.865	4.113
		3	0.567	0.957	1.277	1.915	2.163	2.660	2.943	3.156	3.440	3.688
		1	–	0.452	1.156	2.212	2.966	3.570	4.223	4.776	5.128	5.631
	100	2	0.151	0.553	0.855	1.508	2.011	2.464	2.866	3.519	3.720	4.022
		3	0.402	0.654	1.106	1.408	1.508	1.709	1.911	2.212	2.363	2.614
		1	–	0.251	1.106	1.760	2.212	2.815	3.419	4.072	4.223	4.324
ETH	250	2	0.201	0.754	1.106	1.508	1.810	2.061	2.413	2.916	3.218	3.670
		3	0.452	0.704	1.056	1.257	1.559	1.709	1.810	2.262	2.413	2.614
		1	–	0.302	1.106	1.508	2.011	2.715	3.318	3.670	4.022	4.274
	500	2	0.151	0.603	1.006	1.357	1.609	1.911	2.564	2.916	3.268	3.419
		3	0.553	0.804	1.156	1.408	1.609	1.911	2.011	2.262	2.564	2.866
		1	–	0.551	1.800	3.086	4.115	4.886	5.400	5.841	6.319	6.613
	100	2	0.294	0.882	1.763	2.168	2.902	3.490	3.931	4.262	4.592	5.033
		3	0.441	1.065	1.543	1.763	2.131	2.719	2.866	3.159	3.527	3.931
		1	–	0.367	1.065	2.241	3.049	3.821	4.372	4.996	5.584	6.025
XRP	250	2	0.294	0.551	1.102	1.543	2.094	2.608	3.233	3.600	4.004	4.409
		3	0.514	0.845	1.102	1.396	1.763	2.168	2.388	2.645	3.049	3.306
		1	–	0.331	1.176	2.314	3.086	3.784	4.335	4.666	4.886	5.474
	500	2	0.294	0.698	1.433	2.021	2.388	2.792	3.123	3.380	3.711	3.931
		3	0.698	0.992	1.249	1.616	1.910	2.204	2.535	2.829	3.012	3.123
		1	–	0.502	1.468	2.202	3.090	4.017	4.867	5.292	5.794	6.180
	100	2	0.270	1.004	1.313	1.545	2.317	2.665	3.090	3.438	3.901	4.403
		3	0.541	0.888	1.236	1.622	1.931	2.086	2.317	2.511	2.781	3.090
		1	–	0.309	1.043	1.622	2.626	3.090	3.515	4.056	4.403	4.867
DOGE	250	2	0.193	0.425	1.004	1.043	1.661	2.047	2.472	2.858	3.244	3.438
		3	0.309	0.657	0.811	1.159	1.236	1.545	2.047	2.279	2.433	2.549
		1	–	0.463	1.043	1.893	2.626	3.244	3.940	4.287	4.789	5.176
	500	2	0.154	0.579	1.004	1.545	1.970	2.704	2.897	3.322	3.592	3.901
		3	0.386	0.618	1.081	1.313	1.584	1.931	2.279	2.665	2.974	3.129
		1	–	0.638	1.525	2.801	3.546	4.184	4.929	5.390	5.709	6.099
	100	2	0.461	1.099	1.560	2.340	2.766	3.333	3.688	3.972	4.184	4.539
		3	0.957	1.383	1.844	1.986	2.234	2.553	2.979	3.121	3.440	3.617
		1	–	0.461	1.312	2.270	3.121	3.759	4.078	4.539	5.000	5.355
LTC	250	2	0.213	0.851	1.454	1.915	2.447	2.908	3.156	3.404	3.617	3.936
		3	0.887	1.277	1.525	1.738	2.092	2.376	2.660	2.979	3.227	3.546
		1	–	0.567	1.489	2.518	3.191	3.759	4.255	4.716	5.000	5.390
	500	2	0.284	0.638	1.348	1.809	2.376	2.730	3.333	3.582	3.936	4.255
		3	0.674	1.064	1.454	1.667	1.809	2.234	2.553	2.766	2.979	3.404

Table B3. Continued

Panel B: $\alpha = 95\%$												
Cryptocurrency	W_E	$p\backslash\nu$	1	2	3	4	5	6	7	8	9	10
BTC	100	1	–	0.610	1.028	1.376	1.610	1.837	2.007	2.092	2.191	2.248
		2	0.284	0.631	0.915	1.135	1.255	1.418	1.574	1.631	1.695	1.759
		3	0.447	0.624	0.809	0.908	1.028	1.156	1.241	1.326	1.369	1.397
	250	1	–	0.518	1.021	1.312	1.518	1.702	1.801	1.908	1.986	2.064
		2	0.333	0.645	0.901	1.099	1.284	1.333	1.461	1.553	1.660	1.730
		3	0.582	0.745	0.879	0.972	1.071	1.184	1.284	1.326	1.369	1.475
	500	1	–	0.560	0.950	1.262	1.489	1.652	1.780	1.922	1.979	2.064
		2	0.319	0.638	0.865	1.078	1.241	1.355	1.440	1.532	1.638	1.681
		3	0.489	0.681	0.901	1.050	1.121	1.206	1.277	1.362	1.418	1.461
ETH	100	1	–	0.483	0.985	1.357	1.639	1.820	1.961	2.001	2.031	2.071
		2	0.271	0.623	0.915	1.096	1.227	1.388	1.498	1.549	1.629	1.659
		3	0.412	0.553	0.764	0.915	1.106	1.217	1.297	1.327	1.398	1.468
	250	1	–	0.452	0.875	1.126	1.337	1.468	1.619	1.740	1.790	1.840
		2	0.322	0.483	0.814	0.975	1.076	1.237	1.347	1.378	1.398	1.478
		3	0.412	0.543	0.704	0.875	1.006	1.096	1.176	1.227	1.277	1.317
	500	1	–	0.392	0.714	1.106	1.267	1.368	1.518	1.609	1.699	1.770
		2	0.282	0.513	0.734	0.975	1.096	1.176	1.217	1.297	1.398	1.508
		3	0.422	0.593	0.784	0.905	0.945	1.026	1.136	1.166	1.217	1.247
XRP	100	1	–	0.654	1.198	1.506	1.712	1.925	2.101	2.226	2.381	2.483
		2	0.404	0.698	0.985	1.168	1.381	1.521	1.624	1.712	1.771	1.888
		3	0.441	0.698	0.904	1.029	1.139	1.278	1.367	1.447	1.528	1.587
	250	1	–	0.463	0.985	1.301	1.521	1.741	1.925	2.028	2.094	2.197
		2	0.279	0.632	0.918	1.132	1.301	1.447	1.550	1.631	1.793	1.881
		3	0.441	0.610	0.838	1.007	1.139	1.256	1.301	1.418	1.514	1.565
	500	1	–	0.514	0.940	1.271	1.514	1.668	1.844	1.976	2.094	2.182
		2	0.375	0.610	0.896	1.139	1.293	1.411	1.506	1.587	1.690	1.807
		3	0.478	0.661	0.823	0.889	1.043	1.154	1.227	1.330	1.403	1.462
DOGE	100	1	–	0.479	1.004	1.383	1.591	1.815	1.962	2.078	2.202	2.325
		2	0.355	0.618	0.850	1.081	1.290	1.414	1.537	1.669	1.761	1.869
		3	0.379	0.525	0.718	0.873	1.004	1.105	1.228	1.306	1.421	1.483
	250	1	–	0.386	0.772	1.112	1.429	1.684	1.823	1.908	2.039	2.148
		2	0.239	0.510	0.765	0.942	1.166	1.344	1.445	1.584	1.692	1.738
		3	0.363	0.564	0.711	0.834	0.958	1.089	1.159	1.236	1.313	1.475
	500	1	–	0.417	0.796	1.128	1.360	1.530	1.684	1.823	1.916	2.055
		2	0.286	0.518	0.788	0.973	1.128	1.282	1.421	1.576	1.653	1.738
		3	0.417	0.541	0.749	0.919	0.997	1.112	1.221	1.298	1.367	1.429
LTC	100	1	–	0.631	1.050	1.383	1.574	1.709	1.830	1.957	2.028	2.113
		2	0.376	0.688	0.929	1.121	1.284	1.397	1.475	1.546	1.645	1.723
		3	0.496	0.695	0.780	0.908	1.000	1.113	1.206	1.291	1.340	1.418
	250	1	–	0.539	0.943	1.227	1.426	1.546	1.667	1.709	1.794	1.858
		2	0.340	0.645	0.830	1.057	1.191	1.355	1.447	1.511	1.560	1.617
		3	0.511	0.709	0.894	0.986	1.113	1.177	1.248	1.312	1.376	1.462
	500	1	–	0.560	0.979	1.234	1.433	1.553	1.688	1.794	1.894	1.979
		2	0.312	0.645	0.915	1.106	1.241	1.355	1.482	1.546	1.567	1.638
		3	0.461	0.660	0.844	0.993	1.106	1.234	1.291	1.369	1.390	1.404

Table B3. Continued

Panel C: $\alpha = 90\%$

Cryptocurrency	W_E	$p\backslash\nu$	1	2	3	4	5	6	7	8	9	10
BTC	100	1	–	0.550	0.894	1.089	1.177	1.262	1.365	1.418	1.479	1.493
		2	0.369	0.610	0.773	0.908	0.961	1.043	1.110	1.142	1.167	1.202
		3	0.454	0.567	0.688	0.752	0.826	0.901	0.940	0.972	1.011	1.046
	250	1	–	0.535	0.816	0.965	1.092	1.170	1.241	1.316	1.340	1.372
		2	0.372	0.596	0.748	0.869	0.943	0.996	1.028	1.082	1.117	1.149
		3	0.475	0.582	0.674	0.762	0.830	0.872	0.926	0.965	1.014	1.046
	500	1	–	0.511	0.809	1.025	1.128	1.213	1.273	1.337	1.390	1.429
		2	0.351	0.596	0.755	0.858	0.968	1.046	1.096	1.145	1.177	1.206
		3	0.500	0.635	0.720	0.805	0.869	0.943	0.989	1.046	1.043	1.060
ETH	100	1	–	0.543	0.865	1.036	1.146	1.217	1.297	1.342	1.373	1.433
		2	0.367	0.588	0.734	0.845	0.915	0.980	1.041	1.071	1.106	1.146
		3	0.437	0.583	0.699	0.774	0.850	0.885	0.920	0.960	0.975	1.041
	250	1	–	0.498	0.729	0.920	1.046	1.116	1.171	1.197	1.252	1.272
		2	0.307	0.578	0.664	0.774	0.835	0.940	0.985	1.051	1.086	1.106
		3	0.442	0.588	0.654	0.764	0.820	0.875	0.925	0.965	0.990	1.016
	500	1	–	0.407	0.709	0.855	0.960	1.076	1.126	1.166	1.207	1.237
		2	0.307	0.563	0.669	0.764	0.865	0.930	0.990	1.021	1.061	1.076
		3	0.463	0.578	0.633	0.749	0.804	0.850	0.900	0.920	0.940	0.970
XRP	100	1	–	0.643	0.940	1.168	1.297	1.444	1.528	1.602	1.657	1.716
		2	0.382	0.621	0.801	0.948	1.062	1.157	1.227	1.260	1.308	1.363
		3	0.503	0.654	0.746	0.823	0.889	0.970	1.007	1.084	1.157	1.198
	250	1	–	0.518	0.860	1.076	1.205	1.319	1.425	1.488	1.543	1.583
		2	0.356	0.632	0.816	0.929	1.058	1.179	1.234	1.304	1.341	1.374
		3	0.500	0.665	0.783	0.882	0.959	1.025	1.073	1.106	1.128	1.165
	500	1	–	0.507	0.830	1.047	1.187	1.304	1.392	1.458	1.521	1.528
		2	0.338	0.632	0.786	0.926	1.025	1.102	1.176	1.227	1.282	1.337
		3	0.467	0.595	0.713	0.812	0.885	0.940	1.014	1.076	1.109	1.176
DOGE	100	1	–	0.533	0.830	1.078	1.232	1.390	1.472	1.553	1.603	1.665
		2	0.309	0.595	0.800	0.892	1.012	1.097	1.170	1.205	1.278	1.321
		3	0.371	0.541	0.668	0.765	0.877	0.935	0.997	1.035	1.078	1.124
	250	1	–	0.429	0.776	1.024	1.136	1.259	1.352	1.433	1.491	1.533
		2	0.297	0.579	0.749	0.865	0.973	1.058	1.116	1.194	1.244	1.294
		3	0.421	0.560	0.684	0.800	0.904	0.985	1.047	1.089	1.116	1.151
	500	1	–	0.429	0.749	0.919	1.151	1.267	1.344	1.394	1.445	1.483
		2	0.321	0.556	0.726	0.873	0.985	1.070	1.147	1.190	1.228	1.275
		3	0.444	0.583	0.688	0.769	0.850	0.946	1.035	1.062	1.109	1.143
LTC	100	1	–	0.571	0.840	1.035	1.160	1.262	1.340	1.394	1.457	1.525
		2	0.408	0.631	0.766	0.876	0.975	1.018	1.060	1.121	1.174	1.188
		3	0.472	0.564	0.684	0.738	0.798	0.869	0.904	0.950	1.007	1.053
	250	1	–	0.511	0.773	0.922	1.057	1.142	1.252	1.294	1.330	1.362
		2	0.362	0.571	0.734	0.855	0.922	1.014	1.089	1.131	1.160	1.184
		3	0.518	0.621	0.723	0.812	0.858	0.901	0.965	1.004	1.028	1.050
	500	1	–	0.518	0.741	0.957	1.096	1.160	1.230	1.305	1.362	1.404
		2	0.351	0.603	0.745	0.848	0.933	0.972	1.039	1.074	1.113	1.160
		3	0.450	0.599	0.727	0.773	0.830	0.887	0.933	0.968	1.007	1.046

Table B3. Continued

Panel D: $\alpha = 99.9\%$												
Cryptocurrency	W_E	$p\backslash\nu$	1	2	3	4	5	6	7	8	9	10
BTC	100	1	–	1.773	3.191	7.092	12.766	17.021	22.340	24.823	26.596	29.787
		2	1.064	2.482	2.837	4.255	6.383	8.511	12.411	13.830	17.376	19.504
		3	2.128	2.837	3.901	3.901	5.674	7.447	10.284	12.766	13.830	16.312
	250	1	–	1.064	2.482	4.610	8.156	11.702	15.603	17.021	20.567	23.050
		2	0.709	1.418	2.482	5.319	6.738	8.156	10.284	12.411	15.248	16.312
		3	1.773	2.128	3.191	6.738	7.801	8.156	9.929	10.638	13.830	15.248
	500	1	–	0.709	3.191	4.610	8.156	13.475	16.312	19.149	22.340	25.177
		2	1.064	1.418	1.773	4.255	6.383	7.447	9.220	12.057	14.894	18.085
		3	1.064	1.773	2.837	4.610	5.674	7.092	8.156	8.865	12.057	13.830
ETH	100	1	–	1.006	2.011	5.028	8.044	9.553	12.066	16.088	19.105	21.619
		2	0.503	1.006	2.011	4.022	5.028	5.530	7.541	9.553	11.564	12.569
		3	0.503	1.508	2.011	2.514	4.525	5.530	8.547	9.050	9.553	10.055
	250	1	–	0.503	1.508	2.011	7.541	10.055	12.569	14.077	15.586	16.591
		2	0.000	0.503	1.006	3.017	6.536	9.050	9.050	11.061	11.564	11.564
		3	0.503	0.503	3.017	4.525	5.530	5.530	7.039	8.547	9.553	9.553
	500	1	–	0.503	2.011	3.519	6.536	8.547	10.055	12.569	13.072	15.083
		2	0.503	0.503	1.508	2.514	3.519	6.033	7.541	9.050	11.061	11.061
		3	0.503	1.006	1.006	3.017	4.022	4.525	7.039	7.541	9.553	10.558
XRP	100	1	–	0.367	1.470	5.878	11.021	15.797	19.471	23.512	27.553	30.860
		2	0.000	1.102	2.204	6.613	8.450	11.021	13.226	14.695	16.165	19.104
		3	0.735	1.102	2.572	4.041	4.776	8.082	10.654	11.756	12.858	13.960
	250	1	–	0.000	1.102	4.409	5.878	9.184	15.430	17.267	18.736	21.675
		2	0.367	0.735	2.572	4.409	5.143	6.613	9.552	9.552	11.756	13.593
		3	1.470	1.837	2.572	3.306	5.511	6.613	8.082	9.184	10.654	12.491
	500	1	–	0.367	1.837	4.041	7.348	10.654	14.328	19.838	23.145	26.451
		2	0.367	1.470	2.572	4.041	6.245	8.450	11.021	12.491	15.430	18.369
		3	1.102	1.470	2.939	4.776	6.245	7.348	8.450	11.389	11.389	12.858
DOGE	100	1	–	0.772	2.317	5.021	8.111	11.974	15.064	19.312	22.789	25.879
		2	0.772	1.931	3.090	5.794	7.725	9.656	11.201	13.132	13.519	15.450
		3	1.545	1.931	3.090	3.862	5.794	6.566	7.339	8.884	11.201	12.360
	250	1	–	0.386	1.545	3.862	7.339	10.429	12.746	15.450	16.609	20.085
		2	0.772	1.159	1.931	2.704	4.249	7.339	8.497	8.884	10.815	11.201
		3	1.931	1.931	1.931	3.476	3.862	6.566	6.952	7.725	9.270	9.270
	500	1	–	0.772	1.545	4.635	8.111	10.815	14.677	16.995	19.699	22.016
		2	0.772	0.772	2.704	3.476	4.635	6.566	8.497	11.201	11.974	14.291
		3	0.772	2.317	2.704	3.476	4.249	5.021	6.952	8.884	9.270	10.815
LTC	100	1	–	1.064	3.091	6.383	11.348	15.603	20.213	23.404	26.950	28.723
		2	0.355	2.482	4.255	6.738	9.574	13.121	15.248	17.730	19.504	20.922
		3	1.064	3.546	6.383	7.801	10.284	11.702	13.830	14.184	15.248	16.312
	250	1	–	0.000	2.482	4.255	9.574	14.539	15.957	20.213	23.050	24.468
		2	0.355	1.418	2.837	3.901	8.156	10.284	11.702	13.830	16.312	18.085
		3	1.418	2.482	3.901	6.028	8.865	9.929	11.702	12.766	13.475	14.894
	500	1	–	0.355	2.482	5.319	8.865	13.830	16.312	21.986	24.468	25.887
		2	0.000	1.064	2.482	2.837	6.383	9.220	10.993	12.411	14.894	17.021
		3	0.709	1.418	2.837	5.319	6.028	8.511	9.574	11.348	12.411	13.475