Exponential Growth Bias under Uncertainty

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Abstract

Accurate understanding of exponential growth is essential for long-term investments. However, individuals often exhibit exponential growth bias (EGB)—a tendency to underestimate exponential growth—which leads to systematic misperceptions of long-term investment outcome distributions. This study investigates these misperceptions and proposes a descriptive theory of how people perceive such distributions. In a within-subjects study, 130 participants allocated investments across four time horizons and provided incentivized estimates of investment outcome distributions. Results show that, in certain dimensions, the perception of people can be described as if they compound annual return distributions correctly, but over a shorter time horizon than would be correct. This leads to underestimation of expected values and standard deviations, and overestimation of loss probabilities. Importantly, these estimates are significantly correlated both between and within time horizons, suggesting a consistent perceptual pattern, highlighting the importance of addressing EGB to support better long-term financial decision-making. (JEL G41)

Keywords. Exponential growth bias, long-term investment, portfolio choice, return perception, behavioral finance.

1. Introduction

How do investors make long-term investment decisions to save for some event far in the future, such as retirement? A rational expected utility maximiser will compute the long-term outcome distribution of his investment and calculate the expected utility for different amounts of his wealth invested in the portfolio. The investor then chooses the optimal investment amount that maximizes his utility. However, an actual human decision maker may have difficulties in computing the long-term outcome distribution of his investment.

One potential error an investor could make is to underestimate the exponential growth of his investment portfolio (Wagenaar & Sagaria, 1975, p. 419). Several studies (Anderson and Settle, 1996; Benartzi and Thaler, 1999) found that investors are not able to correctly predict return distributions at the planning horizon. Hence, investors have different portfolio allocations when faced with either short- or long-term information about a risky asset. For instance, Benartzi and Thaler (1999, pp. 375 ff.) showed subjects either the distribution of 30-year or 1-year returns. They found that risk taking significantly increases in the 30-year treatment. Anderson and Settle (1996, pp. 346 ff.) also found that investments in a risky asset are higher when the 10-year returns are shown compared to the 1-year returns. The authors argued that individuals lack the ability to predict exponential growth and therefore are not able to calculate the return distribution for a long-horizon investment.

However, in both of these studies, participants were only implicitly and not explicitly asked to generate a projection of investment outcomes at the planning horizon. Hence, it remains unclear which aspects in the perception of long-term investment outcomes are biased, why they are biased and how each of these aspects influences portfolio allocation. Understanding the sources of this bias is particularly important given the substantial impact of long-term savings decisions and the potential for policy changes. For example Benartzi and Thaler (1999, pp. 380 f.) suggest providing the probability distribution of a long-term investment to help decision-makers appreciate the effects of statistical aggregation. However, providing graphical displays of probability distributions may still introduce bias. For instance, Ibrekk and Morgan (1987, pp. 522 ff.) show that even when individuals are explicitly asked to report the expected value, they often report the mode instead when presented with right-skewed probability density functions, which are typical for long-term investments. These findings highlight the importance of identifying which components of long-term investment outcome distributions are misperceived. A clearer understanding of these biases could inform targeted interventions—such as explicitly presenting distributional features of long-term investments like the expected value, standard deviation, skewness, and loss probability—to help correct individual misperceptions and improve long-term financial decision-making.

The results indicate that individuals exhibit a systematic bias affecting multiple aspects of how they perceive long-term investment outcome distributions. In particular, distortions in the expected value, standard deviation, and loss probability appear to stem from a single underlying bias rather than from independent errors in each dimension. First, individuals underestimate the expected value, standard deviation and skewness and overestimate the loss probability. Second, the expected value, standard deviation and loss probability are consistently over- or underestimated across investment horizons Third, an underestimation in the expected value is accompanied by an underestimation in the standard deviation and an overestimation in the loss probability. Regarding portfolio choice, both the subjective expected value and subjective loss probability influence risk taking significantly. While previous research has focused on the role of objective loss probabilities, the present findings also indicate that the subjective loss probability plays a critical role, especially in contexts where objective probabilities are unavailable or difficult to assess.

2. Background and Hypotheses

One factor that might explain the incorrect perception of long-term investment outcome distributions is the exponential growth bias (EGB), which is the tendency of individuals to systematically underestimate the growth of exponential processes (Stango & Zinman, 2009, p. 2818). Consider a savings scenario, where a present value (PV) is invested at a known periodic interest rate i over time horizon t. The future value (FV) is then calculated as follows:

$$FV = PV * (1 + i)^t. \quad (1)$$

Because the term $f(i, t) = (1 + i)^t$ is the exponential function that is commonly underestimated by individuals, Stango and Zinman (2009, p. 2819) have suggested to incorporate the parameter θ into the relation between present and perceived final value to parametrize the EGB:

$$FV^{perceived} = PV * (1 + i)^{(1-\theta)t}. \quad (2)$$

A correct answer is given for $\theta = 0$. Participants with typical EGB have a $\theta > 0$ and provide a FV that underestimates the effect of compound interest for a given PV, t and i. It is important to note that (2) is a descriptive theory because it describes how individuals behave and not how they actually calculate the FV of an investment. Consider an investor that linearizes compound interest and beliefs that $\$1{,}000$ invested at 10% interest rate for 20 years will be worth $1{,}000 * (1 + 0.1 * 20) = 3{,}000$ dollars, which corresponds to a θ of 0.424. The behavior of this investor can be described *as if* he fully appreciated compound interest, but for a shorter time period of $(1 - 0.424) * 20 = 11.52$ instead of 20 years (Foltice & Langer, 2017, p. 173).

The present study proposes an extension of this descriptive theory to describe how individuals perceive long-term investment outcome distributions. Specifically, it is hypothesized that individuals perceive the distribution of outcomes of a long-term investment *as if* they compound a PV with an annual return distribution correctly, but for a shorter time period than would be correct. Consider an investment scenario where a PV is invested in a risky asset with return distribution X for the time horizon t, where the returns are independent between each period. The distribution of future values of the investment (FV_{dist}) is then calculated as follows:

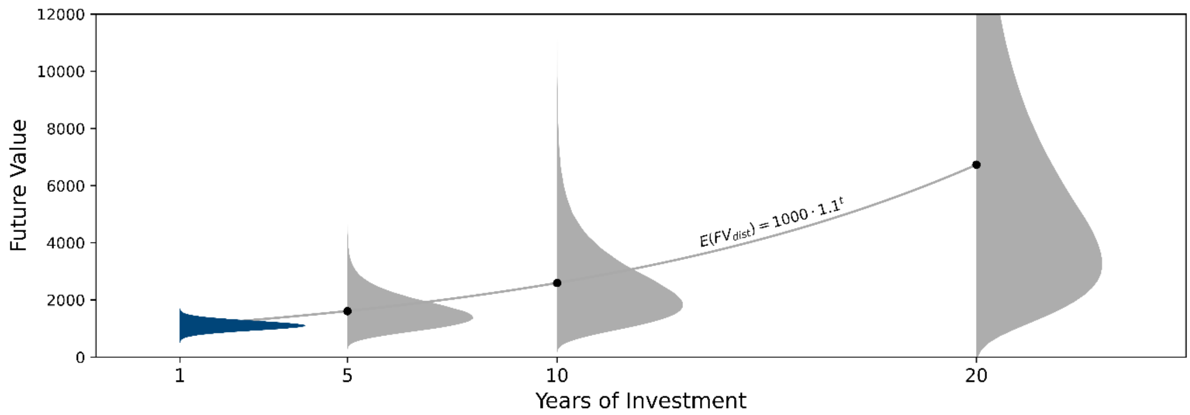
$$FV_{dist} = PV * X^t. \quad (3)$$

Imagine that FV_{dist} refers to an investment of $PV = \$1000$ in an asset with return distribution X, where X follows a normal distribution with a mean return of 10% and a standard deviation of 17%. A rational decision maker investing in that asset, e.g. for 5, 10 and 20 years, would calculate the objective investment outcome distributions depicted in Figure 1. The line depicted in Figure 1 represents the expected value of the investment outcome distributions with respect to the time horizon of the investment. Because the expected return of X is 10%, the expected value of the investment outcome distribution,

$$E(FV_{dist}) = PV * (1+i)^t, \quad (4)$$

will also grow by 10% each year, i.e. $E(FV_{dist}) = 1000 * 1.1^t$ in the example above.

Figure 1: Objective Investment Outcome Distributions.



Fama and French (2018, p. 233) show that FV_{dist} converges toward a lognormal distribution as t increases. This holds true for any distribution X and implies that, as t increases, an investment in an asset with return distribution X will have a higher expected value, a higher standard deviation, a higher skewness, and a lower loss probability (see Appendix A).

Because the term $f(X,t) = X^t$ is the exponential function that is hypothesized to be underestimated by individuals with EGB, it is theorized that the perceived distribution of investment outcomes can be described by the following function, where the parameter θ parametrizes the EGB of an individual:

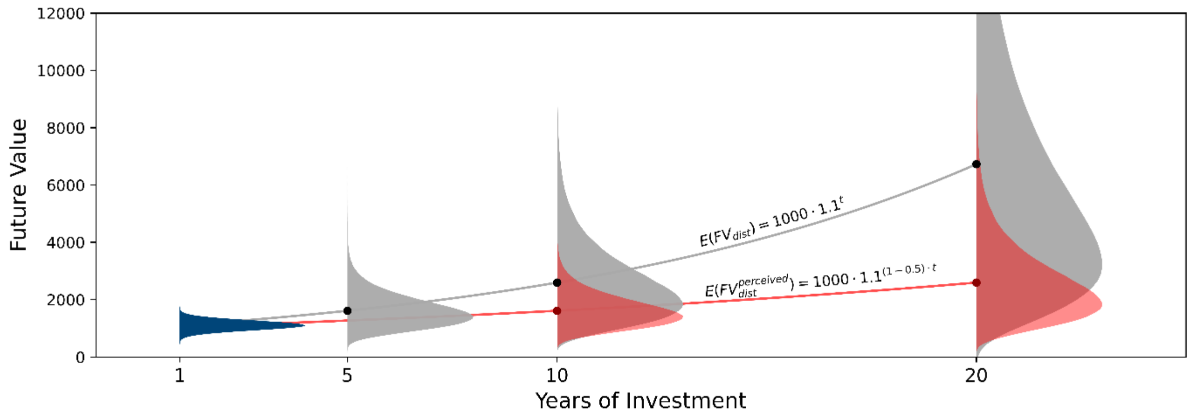
$$FV_{dist}^{perceived} = PV * X^{(1-\theta)t}. \quad (5)$$

Each individual is theorized as having a single EGB parameter θ, which applies consistently across time horizons. Consider again an investment of $PV = \$1000$ in an asset with normally distributed returns X (mean = 10%, SD = 17%). However, the investor now is no longer assumed to be fully rational. Instead, he is assumed to possess an EGB characterized by a parameter of $\theta = 0.5$. This implies that the investor evaluates the investment *as if* compounding occurred over $(1-\theta)t$ rather than the full time horizon t. As a result, the perceived distribution of the investment outcomes is $FV_{dist}^{perceived} = 1000 * X^{(1-0.5)t}$. Figure 2 shows the perceived distributions in red, with 10 and 20 years seen as 5 and 10 years. Notably, the perceived 10-year distribution in Figure 2 corresponds exactly to the objective 5-year distribution. The line in Figure 2 reflects the perceived expected values, which under EGB are given by

$$E\left(FV_{dist}^{perceived}\right) = PV * (1+i)^{(1-\theta)t} \quad (6)$$

with $i = E(X)$. In the present example, $E\left(FV_{dist}^{perceived}\right) = 1000 * 1.1^{(1-0.5)t}$ is depicted as the grey line in Figure 2 and lies below the objective expected value line from Figure 1.

Figure 2: Subjective Investment Outcome Distributions of an Investor with an EGB characterized by $\theta = 0.5$.



For the present study, only investment horizons higher than one are deemed to be relevant, as individuals are assumed to understand that at least the investment outcome distribution based on the annual return distribution will apply over the investment horizon.

To summarize, the descriptive theory outlined above predicts that individuals perceive long-term investments *as if* they compound annual return distributions correctly, but over a shorter time horizon than would be correct. It is further hypothesized that the exponential growth bias parameter θ is constant for each individual, such that the degree of underestimation applies uniformly across different time horizons. This misperception is expected to produce systematic errors in the properties of long-term investment outcome distributions:

First, individuals are expected to consistently *underestimate* the expected value, standard deviation, and skewness, while *overestimating* the loss probability of long-term investments.

If such misperceptions are present, they are likely to influence investment decisions. Research by Nosić and Weber (2010) shows that risk taking depends on perceived return, risk attitude, and risk perception. Moreover, Holzmeister et al. (2020) demonstrate that the loss probability significantly influences risk perception. Thus, a higher perceived return should be associated with greater risk taking, while a higher perceived loss probability should correspond to lower investment in risky assets.

Second, if individuals behave in accordance to the descriptive theory, their estimates should show consistency *between* investment horizons. For example, the subjective expected value of a 10-year investment should be positively correlated with the subjective expected value of a 20-year investment.

Third, the descriptive theory also implies internal consistency *within* each investment horizon. For example, the subjective expected value of a 10-year investment should be negatively correlated with the subjective loss probability of a 10-year investment.

Finally, this study examines the extent to which EGB reflects a general cognitive bias across financial domains. Since the mathematical form of EGB regarding the expected value is identical in both riskless and risky investments (i.e., Equations 2 and 6), it is expected that individuals exhibit a consistent bias across these two domains. Furthermore, this study connects to findings by Stango and Zinman (2009), who also examine misperceptions of exponential growth, but on the borrowing side in the form of payment/interest bias - defined as the systematic tendency to underestimate a loan's interest rate when given the principal, monthly payment, and maturity. A positive correlation between the EGB from Equations 2 or 6 and the payment/interest bias could allow for a transfer of results between studies and across financial domains.

3. Study Setup

3.1 Study Design

Overview. Participants first answered questions assessing their financial literacy. Then they received information about a risk-free and a risky asset, presented alongside an experience sampling design. Thereafter, subjects were asked to evaluate the perceived riskiness of multiple long-term investments in the risky asset, followed by a series of long-term investment tasks, where an initial endowment had to be allocated between the two assets. Afterward, the subjective expected value and subjective loss probability for several long-term investments in the risky asset were elicited. Next, participants provided estimates of the probability density function for these investments. Following this task, participants were given the opportunity to update their perceived riskiness and investment allocation from the previous investments. The same updating procedure was repeated after revealing the true distribution of investment outcomes for multiple long-term investments in the risky asset. Finally, the EGB under certainty and payment/interest bias was assessed, followed by a cognitive reflection test. Participants' willingness to take financial risks was then recorded alongside demographic information.[a]

Financial Literacy. The questions by van Rooij, Lusardi and Alessie (2011, pp. 452 ff.) were used to measure basic (five questions) and advanced (eleven questions) financial literacy. Basic and advanced financial literacy are then defined as the sum of correct answers.

Experience Sampling Task. Participants were introduced to a risk-free and a risky asset. The risk-free asset pays no return and the return of the risky asset follows a normal distribution with an expected return of 10% and a standard deviation of 17% per period. These parameters were explicitly communicated to participants. These parameters were conveyed through an experience-sampling design adapted from Kaufmann, Weber and Haisley (2013, p. 338). Participants were required to sample at least 100 returns from the underlying theoretical distribution. The first 20 returns had to be drawn individually using a “Draw 1 new annual return” button, while the remaining 80 could be drawn in sets of 10 using a “Draw 10 new annual returns” button.

Investment Task. Subjects were asked to allocate $1000 between the risky and risk-free asset for each of four investment horizons: 1, 5, 10, and 20 years. Payment for the investment task was made with a probability of 10%. If selected for payment, one of the four investments was randomly chosen with equal probability to calculate the payoff. The final value of the investment was calculated by randomly sampling returns from the risky asset’s distribution for the corresponding time horizon, and participants earned 1 % of the resulting portfolio value.

[a] The tasks regarding the true investment outcome distribution and the assessment of cognitive reflection are not analyzed in this study, but mentioned for transparency.

Prior to the investment task, risk perception was assessed using a scale adapted from Holzmeister et al. (2020, p. 13): “How risky do you perceive an investment of $1000 in the risky asset if you had to invest in the risky asset for ___ years?” (0 = “Not risky at all”, 10 = “Very risky”). Risk-taking behavior was measured with a format based on Nosić and Weber (2010, p. 299): “Now imagine you have an initial wealth of $1000 and you could invest this amount for ___ year(s) in either the risky or the risk-free asset. How much would you invest in each?” (0% = “Total amount in the risk-free asset”; 100% = “Total amount in the risky asset”). The response step size was 0.1 for the risk perception question and 1 percentage point for the risk taking task. Responses were elicited via a horizontal scrollbar, with the scrollbar thumb initially deactivated until first interaction, to mitigate anchoring effects.

In the investment task, investment decisions for all four time horizons were presented on the same page, allowing participants to view and consider all options simultaneously. This design choice is supported by Freeman and Mayraz (2019, pp. 132 ff.), who propose that choices made in isolation provide only limited opportunity to discover underlying preferences, implying that presenting multiple options at once might lead to more representative decisions.

Subjective expected value of the risky asset. After the investment task, participants were asked to indicate the expected value of a $1000 investment in the risky asset after 5, 10, and 20 years. The adapted question format from Kaufmann, Weber and Haisley (2013, p. 337) is “Imagine you invest $1000 in the risky asset for ___ years. What is the expected value of the invested amount after ___ years? The expected value after ___ years is ___ dollars.” Subjects were incentivized for each question using the scoring rule

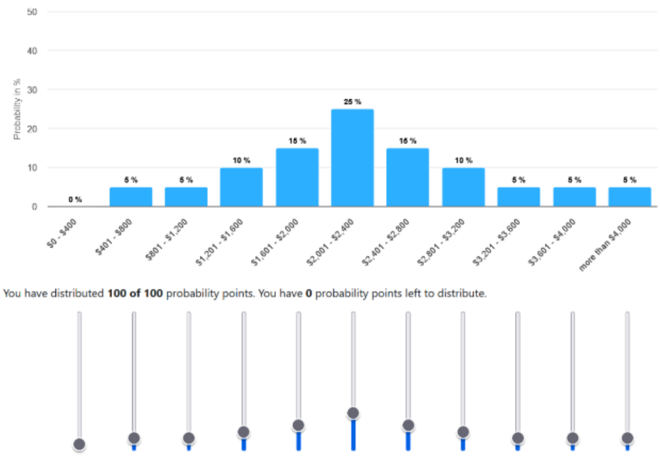
$$Payment = 0.2 - 0.8 * \left(1 - \frac{participants'\,answer}{correct\ answer}\right)^2 \tag{7}$$

The maximum payment of $0.20 was awarded for a perfectly accurate response. Payments decreased with the squared percentage error and reached $0 for absolute percentage errors of 50% or more.

Subjective loss probability of the risky asset. Subjects indicated their estimate for the loss probability of a $1000 investment in the risky asset after 1, 5, 10, and 20 years. The question format based on Kaufmann, Weber and Haisley (2013, p. 337) is “Imagine you invest $1000 in the risky asset for ___ years. What is the probability that the final value of this investment will be less than $1000 after ___ years? The loss probability after ___ years is ___ %.” The inclusion of the 1-year loss probability serves as a check for understanding of the annual return distribution of the risky asset. Participants were incentivized using formula (7).

Slider task: distribution of the risky asset’s final value. Beliefs about the long-term investment outcome distributions of the risky asset were elicited using an adapted version of the “bins and balls” task proposed by Goldstein and Rothschild (2014, p. 13), henceforth called slider task, where participants have to distribute 100 probability points into bins of a specific range (see Figure 3). Beliefs were elicited for the 5, 10 and 20-year return distribution of the risky asset, where the range of the x-axis for the first ten bins is set to 2500, 4000 and 12000, respectively. These limits were chosen to approximate the 90th percentile of each true return distribution and correspond to the 93rd, 87th, and 88th percentiles.

Figure 3: Slider task for the 10-year investment in the risky asset.



Participants were incentivized for each task based on how closely their reported distribution matched the actual distribution, using the overlap measure proposed by Pastore (2019, p. 2):

$$Overlap(A, B) = \sum_{i=1}^{11} \min(A_i, b_i)$$

where $A = (A_1, \dots, A_{11})$ denotes the probability that the participant assigned to each bin and $B = (B_1, \dots, B_{11})$ represents the true probability for each bin. Participants received \$2 for a perfect (100%) overlap, with payment decreasing linearly to \$0 for no (0%) overlap, resulting in a maximum payment of \$6 for all three tasks.

Following the slider tasks, participants were given the opportunity to revise their responses to the earlier risk perception questions and investment allocations, referred to as updated risk perception and updated risk taking.

Exponential growth bias under certainty. The EGB under known interest rate is elicited using the prospective savings question format of Foltice and Langer (2017, p. 183): “You currently have a balance of \$100 in your savings account. You leave this money in your savings account for ___ years at a constant annual interest rate of 10%. “Assume no additional deposits or withdrawals. Interest is compounded annually and reinvested into the account. Based on the above information, the total account balance after ___ years is ___ dollars.” Participants were incentivized using formula (7).

Payment/interest bias. An adapted question format from Stango and Zinman (2009, p. 2813) was used to measure payment/interest bias: "Suppose you want to finance a smartphone priced at $1000 and agree to repay the dealer in 12 equal monthly installments of $100. This means that after 1 year, you will have paid a total of $1200. What annual percent rate of interest do those payments imply?" Participants were also incentivized using formula (7).

Willingness to take financial risks. Participants' willingness to take financial risk was measured using the item from Kaufmann, Weber, and Haisley (2013, p. 336): "Please estimate your willingness to take financial risks.", rated on a Likert scale from 0 (very low willingness) to 10 (very high willingness).

Control Variables. Demographic variables include age, gender, education and income. Education was measured using the following categories: No formal education, high school, college, undergraduate degree (BA/BSc/other), graduate degree (MA/MSc/MPhil/other), and doctorate degree (PhD/other). Stockholding was measured using the question "Do you hold stocks or stock funds (like mutual/exchange-traded fund shares, etc.)?" where respondents selected either "yes" or "no" (Kaufmann et al., 2013, p. 337). Retirement planning was elicited with the question "Have you ever tried to figure out how much you need to save for retirement?", also using "yes" and "no" as response categories (Lusardi & Mitchell, 2011, p. 518).

3.2 Inferred Variables from the Slider Task

The subjective expected value and loss probability of an investment in the risky asset were elicited twice; once with a direct question and once with a slider task. The standard deviation and skewness of the investment outcome distribution of the risky asset were elicited only via the slider task.

The expected value and standard deviation of the distribution from the slider task are measured with the extended Swanson-Megill approximation for continuous random variables proposed by Keefer and Bodily (1983, pp. 597 f.):

$$Expected\ Value\ =\ 0.4 * Q_{50} + 0.3 * (Q_{10} + Q_{90}) \tag{8}$$

$$Standard\ Deviation\ =\ \sqrt{0.4 * (Q_{50})^2 + 0.3 * ((Q_{10})^2 + (Q_{90})^2) - (Expected\ Value\ )^2} \tag{9}$$

where Q_p refers to fractile p of the random variable. Skewness was calculated with the approximation of Groeneveld and Meeden (1984, p. 392):

$$Skewness = \frac{(Q_{90} - Q_{50}) - (Q_{50} - Q_{10})}{(Q_{90} - Q_{10})}. \tag{10}$$

Observations missing the 90th percentile (Q_{90}) could not be included in the analysis. The loss probability was calculated as $P(X < 1000)$, assuming uniform probability density within each bin of the slider task. The EGB related to the subjective expected value of the risky asset is derived from equation (6).

3.3 Calculating $\boldsymbol{\theta}$

The EGB parameter θ can be calculated separately for each estimate of a participant. The objective investment outcome distribution from equation (3),

$$FV_{dist}(\mathrm{t}) \ = PV * X^t,$$

is used with the parameters $PV = 1{,}000$ and $X \sim Normal\ Distribution\ (\mu = 1.1, \sigma = 0.17)$. As mentioned in chapter 2, only values of $t \geq 1$ are deemed to be relevant, as individuals are assumed to understand that at least the investment outcome distribution based on the annual return distribution will apply over the investment horizon.

Because exact formulas exist for the expected value and the standard deviation of FV_{dist} (as shown below), Monte Carlo Simulations are used to calculate skewness and loss probability. Retrospective analysis shows that calculating outcome distributions for values of $t = 1, 2, \dots, 40$ is sufficient to determine θ for all relevant participant estimates (excluding outliers) using 100 repetitions of 10^6 Monte Carlo Simulations for each investment horizon t. This ensures that the standard error for all estimates is ≤ 0.0001, which allows for precise estimation of θ.

To calculate the EGB parameter θ for each relevant estimate, equation (5) is used:

$$FV_{dist}^{perceived}(t) = PV * X^{(1-\theta)t}.$$

Let $M(t)$ denote the true value of a given distributional moment or property of $FV_{dist}^{perceived}(t)$ $\Big($i.e. $Skewness\left(FV_{dist}^{perceived}(t)\right)$ with skewness as defined in equation (10) and $P\big(FV_{dist}^{perceived}(t) < 1000\big)\Big)$ as a function of time horizon t.

The time point t^* at which $M(t)$ matches $\widehat{m}$ is then computed by:

$$t^* = M^{-1}(\widehat{m}). \tag{11}$$

If the subjective estimate $\widehat{m}$ does not match any simulated value exactly, linear interpolation is used between the two nearest points to obtain t^*. For example, if $\widehat{m} = 7\%$ regarding the loss probability for $t = 20$, then the two nearest points $M(8) = 7.49\%$ and $M(9) = 6.38\%$ are used for linear interpolation, yielding $t^* = 9 + (7 - 6.38) * \frac{8-9}{7.49-6.38} = 8.44$.

Once t^* is calculated, θ is computed as follows:

$$\theta = 1 - \frac{t^*}{t}. \tag{12}$$

In the example above, a subjective loss probability of 7% for $t = 20$ corresponds to $\theta = 1 - \frac{8.44}{20} = 0.58$.

To calculate the EGB regarding the subjective expected value, equation (6) is solved for θ:

$$\theta = 1 - \frac{\ln\left(\frac{E\left(FV_{dist}^{perceived}(t)\right)}{1000}\right)}{\ln(1.1)*t} \tag{13}$$

To calculate the EGB regarding the subjective estimate of the standard deviation, the exact formula for the standard deviation of the product of random variables is used (Goodman, 1962, p. 55):

$$\sigma\left(FV_{dist}(\mathrm{t})\right) = \sqrt{(0.17^2 + 1.1^2)^t - 1.1^{2t}} * 1000. \tag{14}$$

As outlined in chapter 2, t is replaced by $(1-\theta) * t$ in equation (13), resulting in:

$$\sigma\left(FV_{dist}^{perceived}(t)\right) = \sqrt{(0.17^2 + 1.1^2)^{(1-\theta)*t} - 1.1^{(1-\theta)*2t}} * 1000. \tag{15}$$

Numerical approximation is performed to calculate θ.

3.4 Participants

In this observational study, 134 participants from the USA were recruited via Prolific. Four participants were excluded, as their answers to the direct questions about the loss probability exceeded 100%, resulting in a final sample size of 130. The study was administered online using oTree (Chen et al. 2016). The sample consists of 65 males and 65 females. Average (median) participant age was 45.31 (44), ranging from 21 to 74 and average (median) income was $71,933.04 ($54,500), ranging from 0 to 695,000. Participants received an average (median) payment of $13.06 ($10.90) with a standard deviation of $11.69, and individual payments ranged from $6.87 to $126.12. The mean (median) completion time was 43.04 (37.64) minutes.

4. Results

4.1 Misperceptions of Investment Outcome Distributions

The results shown in Table 1 provide strong support for the underestimation of the expected value, standard deviation and skewness, and the overestimation of the loss probability for 5-, 10-, and 20-year investments in the risky asset, across both the direct questions and the slider tasks. The estimates in Table 1 also show that the increase in the expected value and standard deviation is recognized by participants, but with a lower increase than would be correct. This does not hold for skewness, as the estimates for the 10- and 20-year investment outcome distributions are not significantly greater than zero based on one-sample t-tests (p-values of 0.34 and 0.09, respectively).

Table 1: Over-/underestimation of all elicited estimates.

Subjective estimates	Question format	Investment Horizon in Years	N	Mean estimate[b]	Estimate Standard Error	Correct Value[c]
Expected value[d]	Direct question	5	126	1340.36***	39.11	1610.51
		10	123	1950.77***	77.79	2593.74
		20	122	3755.43***	221.76	6727.50
	Slider task	5	126	1206.52***	35.06	1610.51
		10	120	1917.78***	56.66	2593.74
		20	117	5395.54***	184.57	6727.50
Standard deviation	Slider task	5	124	348.16***	13.70	570.00
		10	120	547.95***	21.20	1338.28
		20	117	1573.80***	67.14	5225.46
Skewness	Slider task	5	124	0.0553***	0.0201	0.1772
		10	120	0.0078***	0.0191	0.2798
		20	117	0.0195***	0.0144	0.4063
Loss probability	Direct question	5	130	0.2178***	0.0149	0.1241
		10	130	0.1841***	0.0152	0.0545
		20	130	0.1710***	0.0174	0.0124
	Slider task	5	130	0.3461***	0.0255	0.1241
		10	130	0.1781***	0.0191	0.0545
		20	130	0.0613***	0.0106	0.0124

The results also indicate that the underestimation of the expected value is more pronounced in the slider task than in the direct question for shorter time horizons. For example, in the 5-year case, the mean estimate from the slider task ($1206.52) is significantly lower than the estimate from the direct question ($1340.36) (paired t-test, $p = 0.008$). This is accompanied by a substantially higher estimated loss probability of the slider task (35%) compared to the direct question (22%), which is a difference of 12.83 percentage points (paired t-test, $p < 0.001$). While no statistically significant differences are observed in the 10-year case for both the subjective expected value ($p = 0.7$) and the subjective loss probability ($p = 0.6$), the pattern reverses in the 20-year case: participants report a significantly higher expected value in the slider task ($5395.54) compared to the direct question ($3755.43) (paired t-test, $p < 0.001$), and estimate a lower loss probability (6% vs. 17%) (paired t-test, $p < 0.01$).

[b] One-sided t-tests are conducted for the analyses. ***: $p < 0.01$.

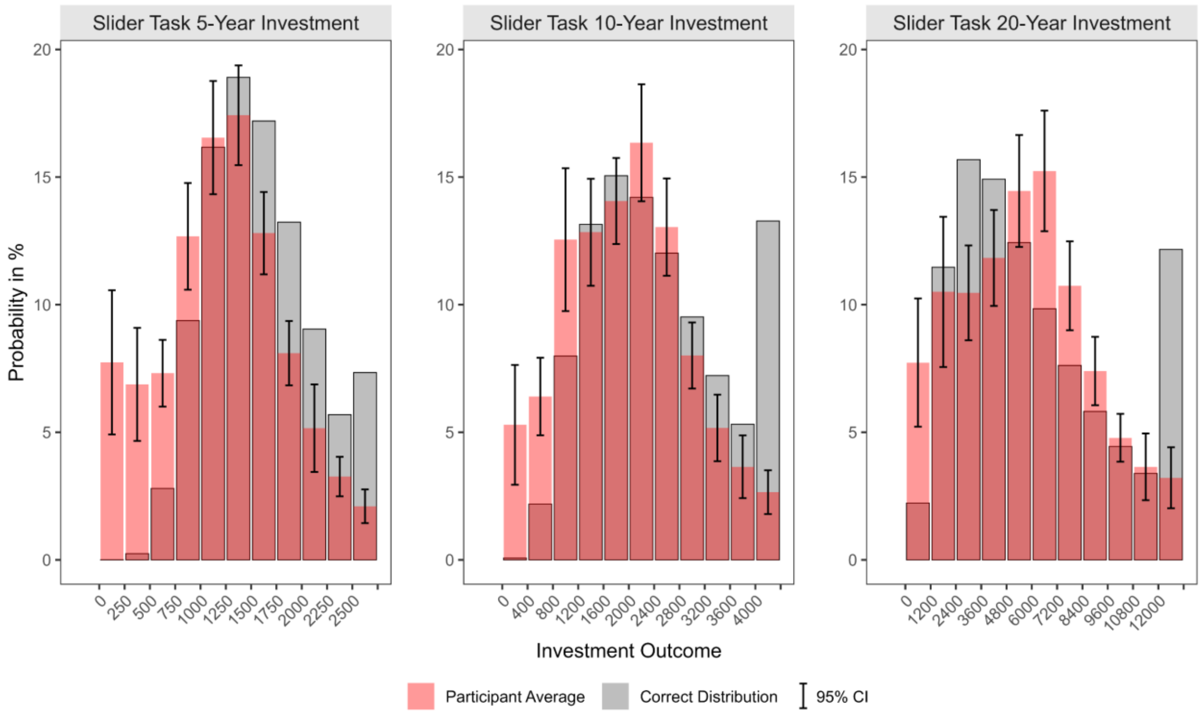
[c] Expected value $= 1000 * 1.1^t$. Standard Deviation $= \sqrt{(0.17^2 + 1.1^2)^t - 1.1^{2t}} * 1000$. Skewness and loss probability were calculated using 10^6 Monte Carlo Simulations of each investment outcome distribution, with skewness calculated as described in equation (10) and loss probability defined as $P(X < 1000)$. This simulation procedure was repeated 100 times to estimate the mean as well as the standard error of each statistic. All standard errors of the Monte Carlo estimates are smaller or equal to 0.0001.

[d] For the direct questions of the expected value, data points above three times the interquartile range were excluded (i.e., values greater than 3139.63, 6175.32, and 21992.50 for the 5-, 10-, and 20-year investment, respectively).

To evaluate whether the underestimation of expected values observed in the slider task could be attributed to features of the task design, it is relevant to consider the upper bounds in the slider task, which were set at $2500, $4000, and $12000 for the 5-, 10-, and 20-year tasks, respectively.

In the 5- and 10-year conditions, the mean expected values reported to the direct question ($1340.36 and $1950.77, respectively) lie approximately midway within the available slider ranges. In these two conditions, the design of the slider task appears to have exerted minimal influence on participants' estimates. In contrast, the broader response range in the 20-year task likely exerted a debiasing effect. The higher upper bound of $12000 may have implicitly encouraged participants to report elevated expected values, thereby exerting upward pressure on their estimates and reducing the degree of underestimation relative to the direct question format.

Figure 4: Average bar height in the slider tasks.



Notes. The last bar corresponds to "more than $2500" (5 years), "more than $4000" (10 years), and "more than $12000" (20 years).

The inability to detect the increase in skewness may be attributable to the representativeness heuristic, which suggests that people assess the likelihood of an event based on how closely it resembles the characteristics of its generating process or reference category (Kahneman & Tversky, 1972). In this context, the one-year return distribution may be used as a reference, and individuals seem to assume that long-term outcomes should exhibit similar statistical properties, such as symmetry. Combined with EGB, the representativeness heuristic may lead individuals to assume that the distribution of long-term investment outcomes resembles that of short-term returns, causing them to overlook how compounding changes the shape of the distribution over time.

4.2 Do Misperceptions influence Risk Taking and Perceived Risk?

According to Nosić and Weber (2010, p. 282), risk taking is a function of perceived return, perceived risk, and individual risk attitude. Table 2 shows the results from several linear regression specifications of the form:

$$Risk\ Taking = \beta_0 + \beta X,$$

where X includes either the subjective expected value or the EGB of the subjective expected value, along with perceived risk, willingness to take financial risks, and a set of control variables. The models in Table 2 also vary by time horizon (10 vs. 20 years) and question format (direct questions vs. slider task).

The results suggest that both the subjective expected value and the EGB in the subjective expected value are negatively associated with risk-taking. Participants who more strongly underestimate the expected value of the risky asset tend to invest less in it.

Table 2: Determinants of Risk-Taking.

Dependent variable	Risk taking				Updated risk taking after the slider task			
Question format	Direct questions				Slider task			
Time horizon	10 Years		20 Years		10 Years		20 Years	
Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Subj. expected value	4.56* (2.53)		1.90* (1.02)		8.08** (3.48)		2.93*** (1.11)	
EGB of the subj. expected value		-4.58** (2.08)		-11.73*** (3.32)		-8.23* (4.34)		-22.53*** (8.22)
Willingness to take financial risks	1.45* (0.73)	1.44** (0.71)	1.60** (0.80)	1.55** (0.76)	1.94** (0.76)	1.79** (0.76)	1.87** (0.79)	1.98** (0.79)
Perceived risk	-0.50*** (0.09)	-0.50*** (0.09)	-0.54*** (0.08)	-0.52*** (0.07)	-0.42*** (0.09)	-0.43*** (0.09)	-0.49*** (0.08)	-0.49*** (0.08)
Income	0.02 (0.03)	-0.01 (0.03)	0.02 (0.03)	-0.01 (0.03)	0.01 (0.03)	0.00 (0.03)	0.02 (0.03)	0.03 (0.03)
Stockholding	6.22 (4.75)	6.65 (4.63)	4.50 (5.23)	6.54 (4.99)	6.36 (4.66)	6.65 (4.69)	7.36 (5.01)	7.32 (5.00)
Retirement planning	2.38 (4.52)	2.03 (4.34)	-2.43 (4.96)	-2.20 (4.66)	-1.81 (4.58)	-1.79 (4.63)	-1.33 (4.93)	-1.17 (4.93)
Basic Financial Literacy	0.19 (2.53)	0.49 (2.42)	0.08 (2.86)	0.06 (2.62)	3.37 (2.57)	3.80 (2.57)	3.90 (2.80)	3.80 (2.79)
Advanced financial literacy	0.53 (1.19)	0.46 (1.14)	0.48 (1.30)	0.14 (1.22)	-0.62 (1.39)	-0.49 (1.40)	-1.30 (1.45)	-1.51 (1.46)
Female	0.14 (3.91)	0.45 (3.76)	-1.13 (4.31)	-0.18 (4.03)	-1.83 (3.99)	-1.41 (4.01)	-0.71 (4.21)	-0.31 (4.19)
Age	0.034 (0.152)	0.022 (0.145)	0.11 (0.17)	0.09 (0.16)	-0.03 (0.15)	-0.03 (0.15)	0.05 (0.16)	0.06 (0.16)
College	-4.10 (5.51)	-4.37 (5.32)	-6.14 (6.04)	-8.34 (5.69)	-0.5 (5.39)	-0.7 (5.43)	-9.87* (5.85)	-10.38* (5.85)
Bachelor's degree	-4.64 (5.07)	-4.16 (4.86)	-5.60 (5.54)	-6.05 (5.22)	-2.61 (4.86)	-2.34 (4.90)	-6.41 (5.29)	-6.61 (5.29)
Master's degree or higher	-12.17* (6.89)	-9.26 (6.44)	-17.77** (7.55)	-16.22*** (6.95)	-9.20 (6.64)	-8.33 (6.66)	-22.10*** (7.10)	-22.15*** (7.08)
Constant	61.69*** (14.49)	73.57*** (15.02)	68.48*** (16.11)	84.97*** (15.89)	50.69*** (15.04)	67.10*** (15.36)	56.84*** (15.67)	77.47*** (15.78)
Observations	123	130	122	130	120	120	117	117
R-squared	0.39	0.39	0.47	0.49	0.38	0.37	0.52	0.52

Adjusted R-squared	0.32	0.32	0.41	0.44	0.30	0.29	0.46	0.46

Holzmeister et al. (2020, p. 10) show that the *objective* loss probability is a central determinant of risk perception. This suggests that the *subjective* loss probability also affects how individuals perceive risk. Table 3 shows the results from several linear regression specifications of the form:

$$Risk\ perception = \beta_0 + \beta X,$$

where X includes subjective estimates of the loss probability, standard deviation and skewness. The models in Table 2 vary by time horizon (10 vs. 20 years) and question format (direct questions vs. slider task).

The results indicate that risk perception is significantly influenced by the subjective loss probability, but not by the other two risk measures.

Table 3: Determinants of Risk Perception.

Dependent variable	Risk perception		Updated risk perception after the slider task	
Question format	Direct Questions		Slider task	
Time horizon	10 Years	20 Years	10 Years	20 Years
Model	(1)	(2)	(3)	(4)
Subj. loss probability	0.38*** (0.11)	0.71*** (0.12)	0.55*** (0.11)	0.77*** (0.12)
Subj. standard deviation			10.92 (8.34)	4.45 (3.42)
Subj. skewness			4.89 (9.16)	20.01 (15.60)
Constant	38.90*** (2.87)	29.51*** (3.04)	26.55*** (5.71)	20.08*** (6.63)
Observations	130	130	120	117
R-squared	0.08	0.24	0.17	0.27
Adjusted R-squared	0.07	0.23	0.15	0.25

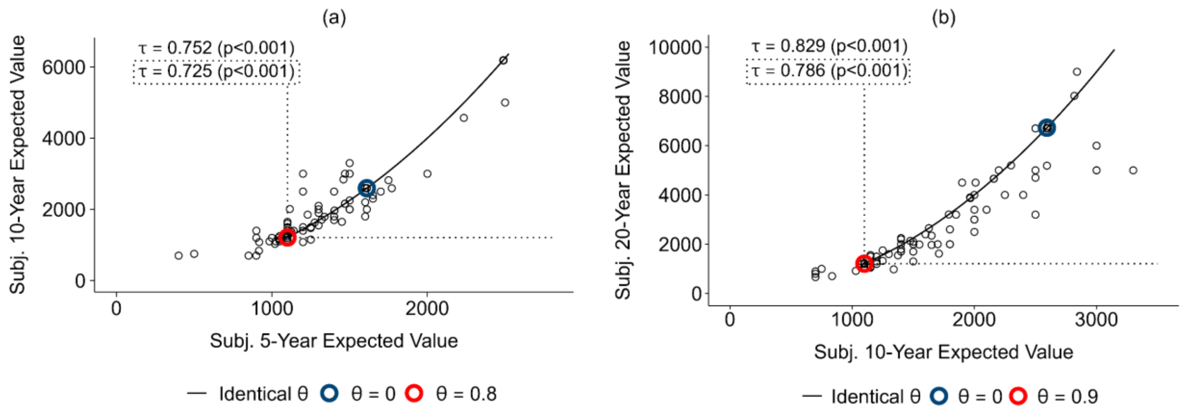
4.3 Consistency in Misperceptions between Investment Horizons

Building on the theoretical framework outlined in chapter 2, this section examines whether misperceptions of investment outcome distributions—specifically regarding the expected value, standard deviation, skewness, and loss probability—are consistent *between* different investment horizons.

To assess this consistency, Kendall's τ is primarily used, as it measures the ordinal association between subjective estimates across investment horizons. Therefore, τ not only captures the degree of association of the raw values, but also indicates whether the underlying parameters θ are consistent across different horizons. As with Pearsons r, τ can take values between +1 and -1, where $\tau = +1$ indicates per

Figure 5 shows the consistency of subjective expected values from the direct questions across investment horizons. Panel (a) presents estimates for the 5- and 10-year investment, while Panel (b) presents estimates for the 10- and 20-year investment. Panels (a) and (b) show that the subjective expected values that can be explained by the descriptive theory are highly correlated across time horizons, with Kendall's τ = 0.725 (p < 0.001) between the 5- and 10-year estimates and τ = 0.786 (p < 0.001) between the 10- and 20-year estimates. However, a subset of estimates—17.7% for the 5-year, 10% for the 10-year, and 11.5% for the 20-year horizon—fall below $1,100, which is the expected value of a one-year investment. These values cannot be accounted for by the model, as they imply a perceived investment period of less than one year.

Figure 5: Consistency Between Investment Horizons regarding the Expected Value (Direct Questions).

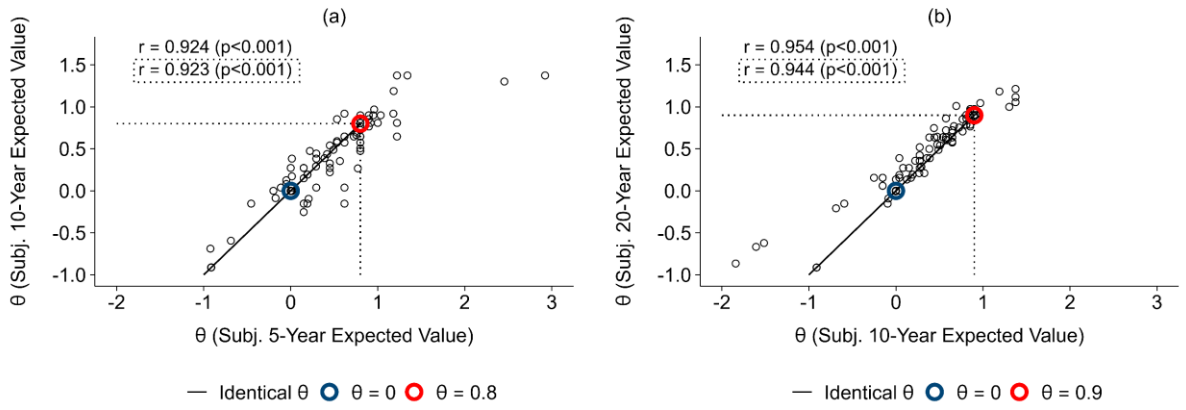


Notes. The dotted box around the value $\tau = 0.725$ $(p < 0.001)$ in Panel (a) and $\tau = 0.786$ $(p < 0.001)$ in Panel (b) highlight Kendall's τ for the subset of participants whose expected values are consistent with the descriptive EGB theory (i.e., for time horizons $t \geq 1$). The other τ values refer to all data points shown in the corresponding graph.

The blue circles in Figure 5 represent a perfectly rational investor reporting expected values of $1000 * 1.1^t$, which correspond to 1610.51 $(t = 5)$, 2593.74 $(t = 10)$, and 6727.50 $(t = 20)$. The red circle in Panel (a) represents an individual whose subjective expected value for a 5-year investment corresponds to the expected value of a 1-year investment, and whose 10-year estimate aligns with the 2-year expected value. This implies that the individual perceives investments as if compounding only occurred over 20% of the true time horizon, which, according to equation (6), corresponds to an EGB of $\theta = 0.8$. Similarly, the red circle in Panel (b) depicts an individual who behaves as if compounding only occurred over 10% of the true time horizon, which corresponds to an EGB of $\theta = 0.9$. The solid line represents estimates that are consistent with a single θ across both investment horizons presented in the graphs.

Figure 6 shows the EGB inferred from the subjective expected values depicted in Figure 5 according to equation (6). The EGB estimates are highly correlated across investment horizons, with Pearson's $r = 0.923$ $(p < 0.001)$ between the 5- and 10-year estimates and $r = 0.944$ $(p < 0.001)$ between the 10- and 20-year estimates that can be explained by the descriptive theory. These results underscore that individuals consistently underestimate the expected value of long-term investments as if compounding occurred over a shortened time horizon.

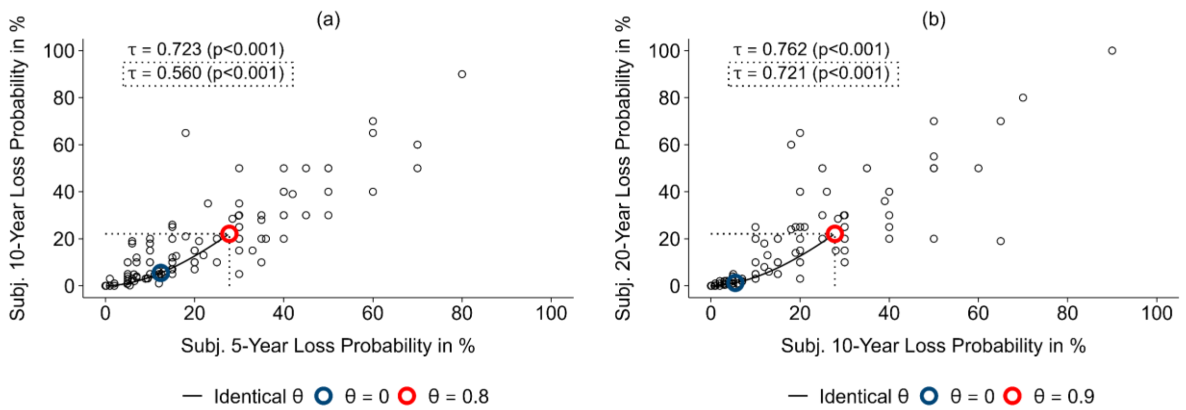
Figure 6: Consistency Between Investment Horizons regarding the EGB of the Subjective Expected Value (Direct Questions).



Notes. For subjective expected values beyond the theoretical bounds indicated by the dotted lines, θ was computed according to equation (6).

Results regarding the subjective loss probability from the direct questions are depicted in Figure 7. The loss probabilities that can be explained by the descriptive theory are highly correlated across time horizons, with Kendall's $\tau = 0.560$ $(p < 0.001)$ between the 5- and 10-year estimates (Panel (a)) and $\tau = 0.721$ $(p < 0.001)$ between the 10- and 20-year estimates (Panel (b)). However, a subset of estimates—34.6% for the 5-year, 26.1% for the 10-year, and 21.5% for the 20-year horizon— exceed the loss probability of 27.8% associated with a one-year investment. These values cannot be accounted for by the model, as they imply a perceived investment period of less than one year.

Figure 7: Consistency Between Investment Horizons regarding the Subjective Loss Probability (Direct Questions).

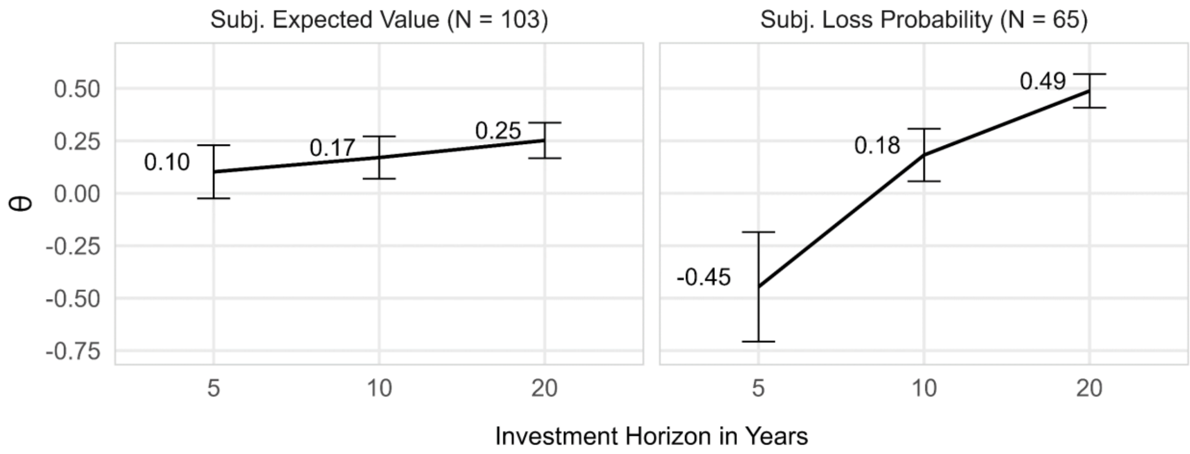


The blue circles represent a perfectly rational investor reporting loss probabilities of 12.41% $(t = 5)$, 5.45% $(t = 10)$, and 1.24% $(t = 20)$. The red circles again represent individuals who behave as if compounding occurred over only 20% of the actual time horizon in Panel (a) and 10% in Panel (b). The dotted lines mark the boundary of what the descriptive theory can account for. Because individuals are assumed to understand that at least the investment outcome distribution based on the annual return distribution will apply over the investment horizon, a loss probability higher than the annual loss probability of 27.8% cannot be explained. Accordingly, the vertical dotted lines reflects the annual loss probability, while the horizontal dotted lines corresponds to the two-year loss probability of 22.1%.

While most estimates are highly correlated between investment horizons, the level of θ varies across estimates of distributional properties. Figure 8 depicts the level of θ for the expected value and loss probability. To ensure comparability within distributional properties, only individuals with a calculable θ for all investment horizons within a given property are included. The results of an ANOVA indicate that differences in estimates of the expected value are not statistically significant ($F_{1,307} = 2.22, p = 0.137$), whereas differences in loss probability estimates are statistically significant ($F_{1,193} = 35.19, p < 0.001$).

Considering the subjective expected value, indiciduals on average behave *as if* they correctly apply compounding over 90% of the time in the 5-year condition, 83% in the 10-year condition, and 75% in the 20-year condition. Considering the subjective loss probability, individuals behave as if compounding applies for 145%, 82%, and 51% of the respective time horizons.

Figure 8: Levels of θ regarding the Subjective Expected Value and Subjective Loss Probability (Direct Questions).



Regarding the expected value, most exclusions occurred because participants provided an estimate below $1100 in at least one condition. Considering the loss probability, most exclusions were due to 52 cases where the estimate exceeded 27.8% and/or 10 cases where it was exactly 0. In all of these cases, θ cannot be calculated.

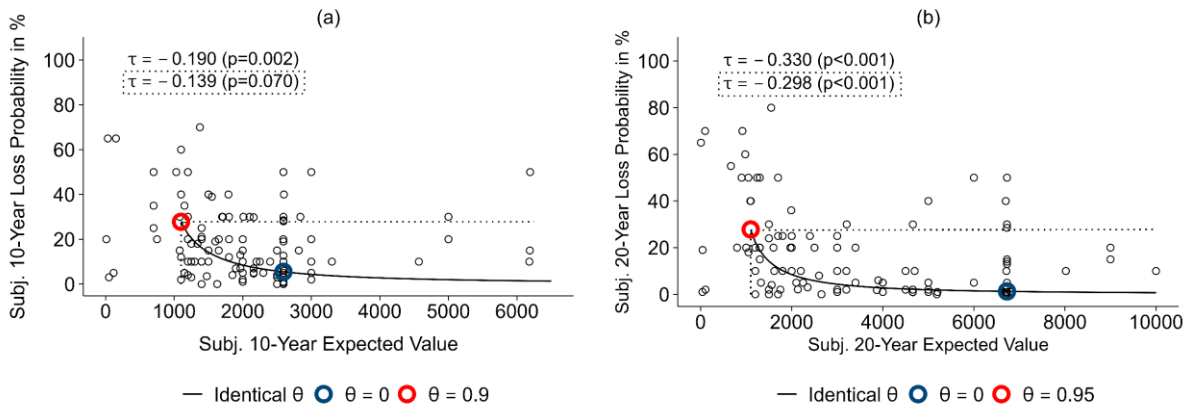
Up to this point, consistency in subjective estimates between investment horizons was examined only for direct questions. Figure B.1.1 in Appendix B shows that, except for skewness, all slider task estimates are significantly correlated between investment horizons, confirming previous findings. Corresponding θ levels are displayed in Figure B.1.2 in Appendix B.

4.4 Consistency in Misperceptions within Investment Horizons

This section examines whether misperceptions of investment outcome distributions—specifically regarding the expected value, standard deviation, skewness, and loss probability—are consistent *within* different investment horizons.

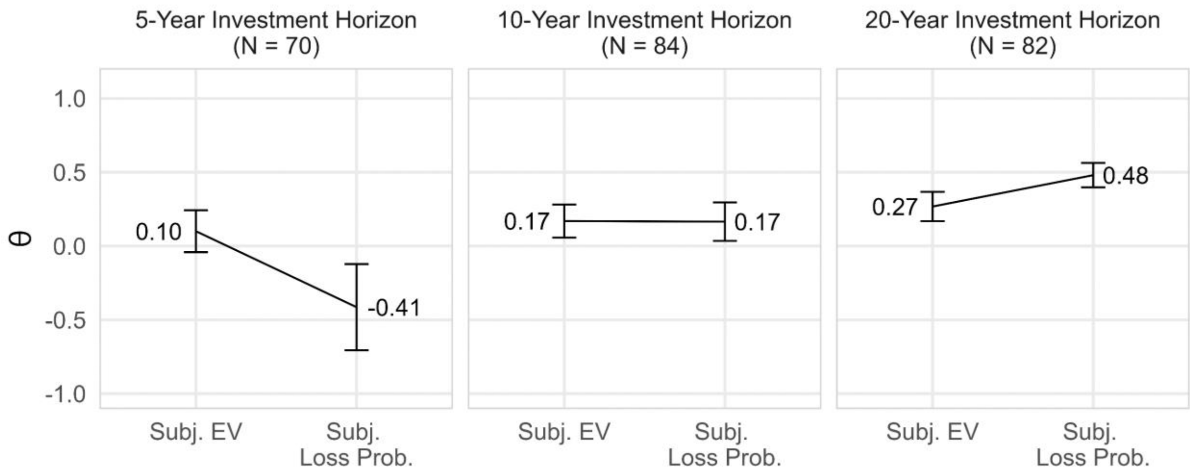
Figure 9 shows the consistency of subjective expected values and subjective loss probabilities derived from the direct questions. Panels (a) and (b) show that the estimates for the expected values and loss probabilities that can be explained by the descriptive theory are correlated within time horizons, with Kendall's $\tau = -0.139\ (p = 0.07)$ between the 10-year estimates and $\tau = -0.298\ (p < 0.001)$ between the 20-year estimates.

Figure 9: Consistency of Subjective Expected Values and Subjective Loss Probabilities Within Investment Horizons (Direct Questions).



To assess the absolute level of θ within each horizon, Figure 10 shows average θ values separately for the expected value and the loss probability regarding the direct questions. Only participants with valid θ estimates for both measures at each time horizon are included, allowing for direct comparison within time horizons across distributional properties. The results of ANOVAs indicate that differences in θ estimates between the expected value and loss probability are statistically significant for the 5-year $(F_{1,138} = 9.66, p = 0.002)$ and 20-year $(F_{1,162} = 10.41, p = 0.002)$ investment horizons. In contrast, no significant difference is observed at the 10-year horizon $(F_{1,166} = 0.002, p = 0.964)$.

Figure 10: Levels of θ regarding different Investment Horizons (Direct Questions).



Notes. Subj. EV = Subjective expected value, Subj. Loss Prob. = Subjective loss probability. Most exclusions from the graph were due to participants estimating loss probabilities above 27.8% (n = 45, 34, 28 for the 5-, 10-, and 20-year horizons, respectively) and/or reporting expected values below \$1100 (n = 23, 13, 15).

As mentioned in Chapter 4.1, Considering the slider tasks, Figure B.2.1 in Appendix B shows that, all slider task estimates are significantly correlated within investment horizons, confirming previous findings. Corresponding θ levels are displayed in Figure B.2.2 in Appendix B.

4.5 EGB under Certainty and Uncertainty

According to equations (2) and (6), the EGB from the subjective expected value of an investment in the risky asset with an uncertain return should be exactly the same as the EGB from the subjective expected value an investment with a certain/fixed return, given the same time horizon and expected return. In other words, the implied θ parameters should be exactly identical.

To test the alternative hypothesis that there is no difference between these two, a robust Bayesian analysis according to Kruschke (2013) is conducted using the direct questions (see Appendix C). The Bayesian approach allows for accepting the null value by defining a region of practical equivalence (ROPE) around the null value, representing parameter values considered negligibly different from the null for practical purposes. For the analysis, a difference of 5% between $FV^{perceived}$ (equation (2)) and $E\left(FV_{dist}^{perceived}\right)$ (equation (6)) – denoted as $\Delta_{\theta,t}$ – is deemed practically equivalent. The corresponding ROPES for θ regarding the 10- and 20-year investment horizon, denoted as $\Delta_{\theta,10}$ and $\Delta_{\theta,20}$, are [-0.054; 0.051] and [-0.027; 0.026], respectively (see Appendix D).

The posterior 95% credible intervals are [0.107; 0.278] for $\Delta_{\theta,10}$ and [0.178; 0.349] for $\Delta_{\theta,20}$, respectively. Since both intervals lie entirely outside their corresponding ROPEs, the results indicate that the EGB parameters θ under certainty and uncertainty differ significantly.

Using Kendall's τ to analyze the correlation between the EGB under certainty and uncertainty for each time horizon, the results reveal a non-significant correlation for the 10-year horizon ($\tau = 0.065, p = 0.296$) and a significant positive correlation for the 20-year horizon ($\tau = 0.153, p = 0.013$). These relatively low correlations—despite the same mathematical structure—suggest that individuals rely on different cognitive processes when estimating exponential growth in certain versus uncertain investment scenarios.

4.6 EGB and Payment/Interest Bias

In this section, the correlation between the answer to the adapted payment/interest bias question from Stango and Zinman (2009, p. 2825) and the EGB under certainty and uncertainty is assessed. The results in Table 4 reveal no statistically significant correlation between the EGB and payment/interest bias using Kendalls τ, suggesting that the two biases may operate independently and are likely driven by distinct underlying cognitive processes.

Table 4: Kendall's Rank Correlation Coefficient Between the EGB in the Subjective Expected Value and the Payment/Interest Bias

	θ of the Subjective Expected Value from the Direct question		θ of the Subjective Expected Value from the Slider Task	
Investment Horizon in Years	10	20	10	20
Response to the Payment/Interest Bias question	-0.080 (-1.155)	-0.086 (-1.228)	0.017 (0.252)	-0.061 (-0.911)
Sample size	117	115	129	130
Note: z-value in parentheses. *p<0.1; **p<0.05; ***p<0.01.				

5. Conclusion

This study proposes and empirically tests a descriptive theory of how individuals perceive long-term investment outcome distributions. The theory posits that individuals exhibit an exponential growth bias (EGB) when evaluating long-term investments: people perceive the growth of an investment outcome distribution *as if* compounding based on the annual return distribution occurred over a shorter time horizon than it actually does.

Empirical evidence supports the central predictions of the proposed descriptive theory. Participants systematically underestimate the expected value and standard deviation while overestimating the probability of losses. These distortions are relatively consistent between and within time horizons. Regarding the consistency between time horizons, the subjective estimates for each individual distributional property—expected value, loss probability and standard deviation—show significant correlations across investment horizons. There is also evidence that the level of EGB inferred from the subjective expected value remains relatively stable across horizons. In contrast, the level of EGB derived from loss probability estimates varies more substantially. Considering the consistency within time horizons, subjective estimates of the expected value are significantly correlated with subjective estimates of loss probability and standard deviation at longer investment horizons. However, the pattern in the level of EGB is mixed, suggesting that the EGB does not manifest uniformly across different distributional properties within a given time horizon.

Interestingly, skewness stands out as a notable exception among the distributional properties. While participants systematically misperceive expected value, standard deviation, and loss probability, skewness is consistently underestimated and not significantly different from zero across time horizons. This indicates that individuals largely fail to recognize the right-skewed nature of long-term investment distributions, suggesting that their mental representation deviates from the lognormal form, which becomes increasingly skewed over time.

The degree of misperception in both expected value and loss probability significantly influences portfolio choices. Individuals who more strongly underestimated expected values tend to allocate less to risky assets. Among the risk-related distributional properties—loss probability, standard deviation, and skewness—only the subjective loss probability has a significant impact on risk perception and subsequent investment behavior. These findings indicate that risk-taking is primarily driven by perceptions of expected value and loss probability. Consequently, targeted interventions that correct EGB in the perceived expected value and loss probability may improve long-term financial decision-making—potentially offering a more practical approach than presenting individuals with the full investment outcome distribution.

Appendix A. Properties of a Lognormal Distribution

Assume a lognormal variable Y with mean μt and standard deviation $\sigma\sqrt{t}$, i.e. $Y \sim LogNormal(\mu t, \sigma\sqrt{t})$, where $\mu > 0$ and $\sigma > 0$ are constants and t denotes time. The moments of Y are (Aitchison & Brown, 1969, p. 8):

$E(Y) = e^{(\mu+\frac{1}{2}\sigma^2)t}$ with $\frac{dE(Y)}{dt} > 0$

$SD(Y) = e^{(\mu+\frac{1}{2}\sigma^2)t}\sqrt{e^{\sigma^2 t} + 1}$ with $\frac{dSD(Y)}{dt} > 0$

$Skew(Y) = (e^{\sigma^2 t} + 2)\sqrt{e^{\sigma^2 t} - 1}$ with $\frac{dSkew(Y)}{dt} > 0$

The probability that Y takes on values less than 1 is given by

$L(Y) = \Phi\left(\frac{\ln(1)-\mu t}{\sigma\sqrt{t}}\right)$ with $\frac{dL(Y)}{dt} = \Phi\left(\frac{\ln(1)-\mu t}{\sigma\sqrt{t}}\right) * \left(-\frac{\mu}{2\sigma\sqrt{t}}\right) < 0$

Appendix B. Supplementary Figures

Figure B.1.1: Consistency Between Investment Horizons regarding estimates of the Slider Tasks.

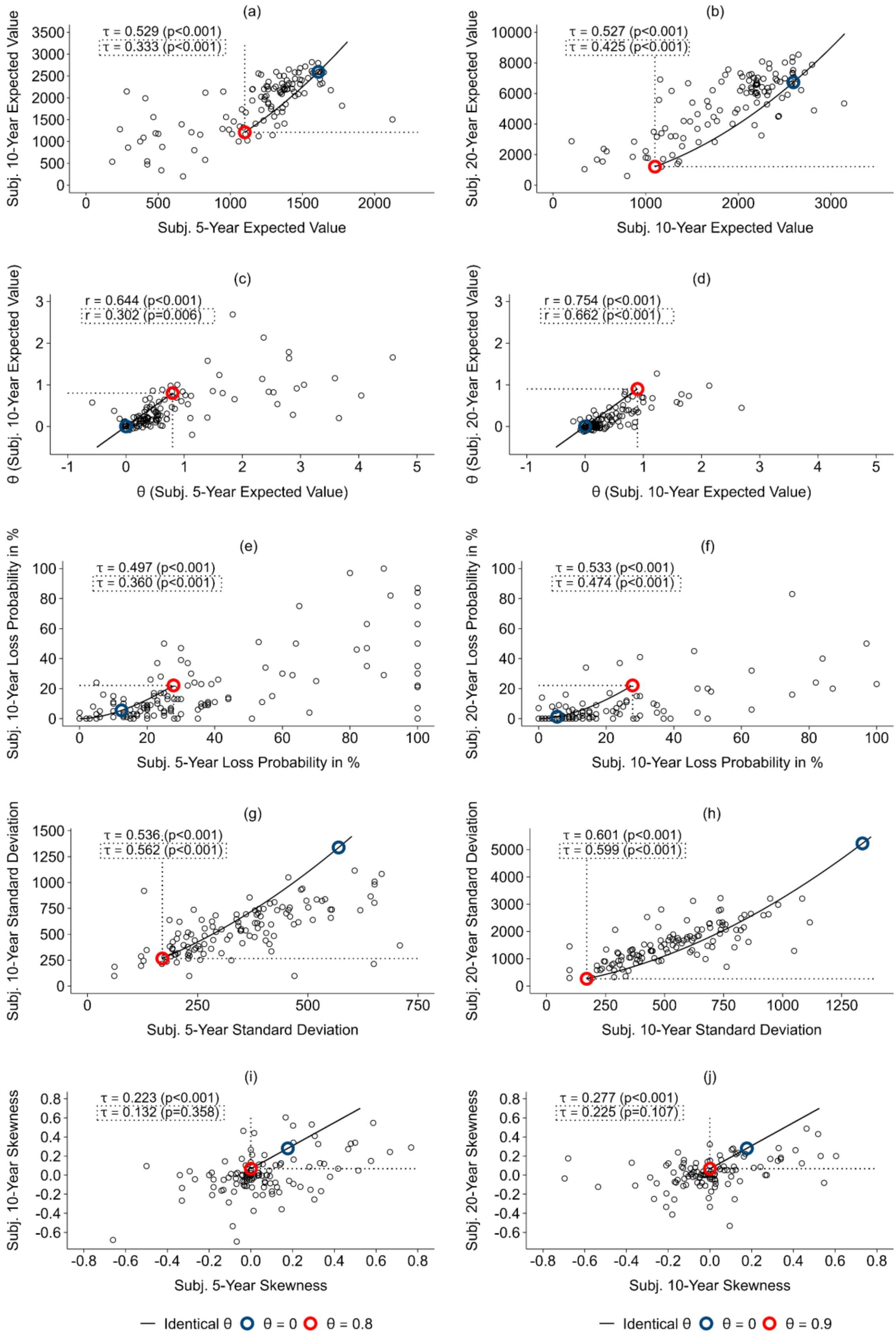
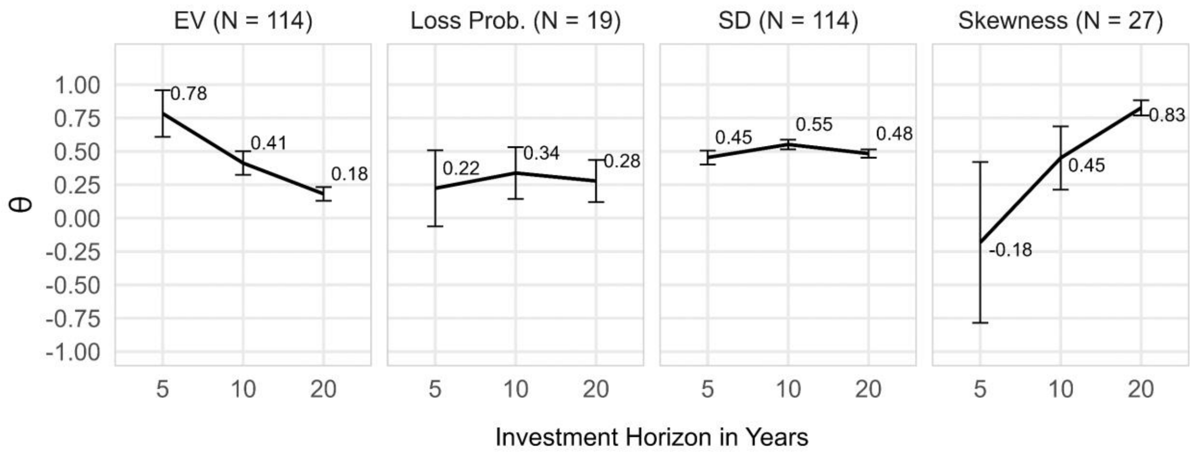
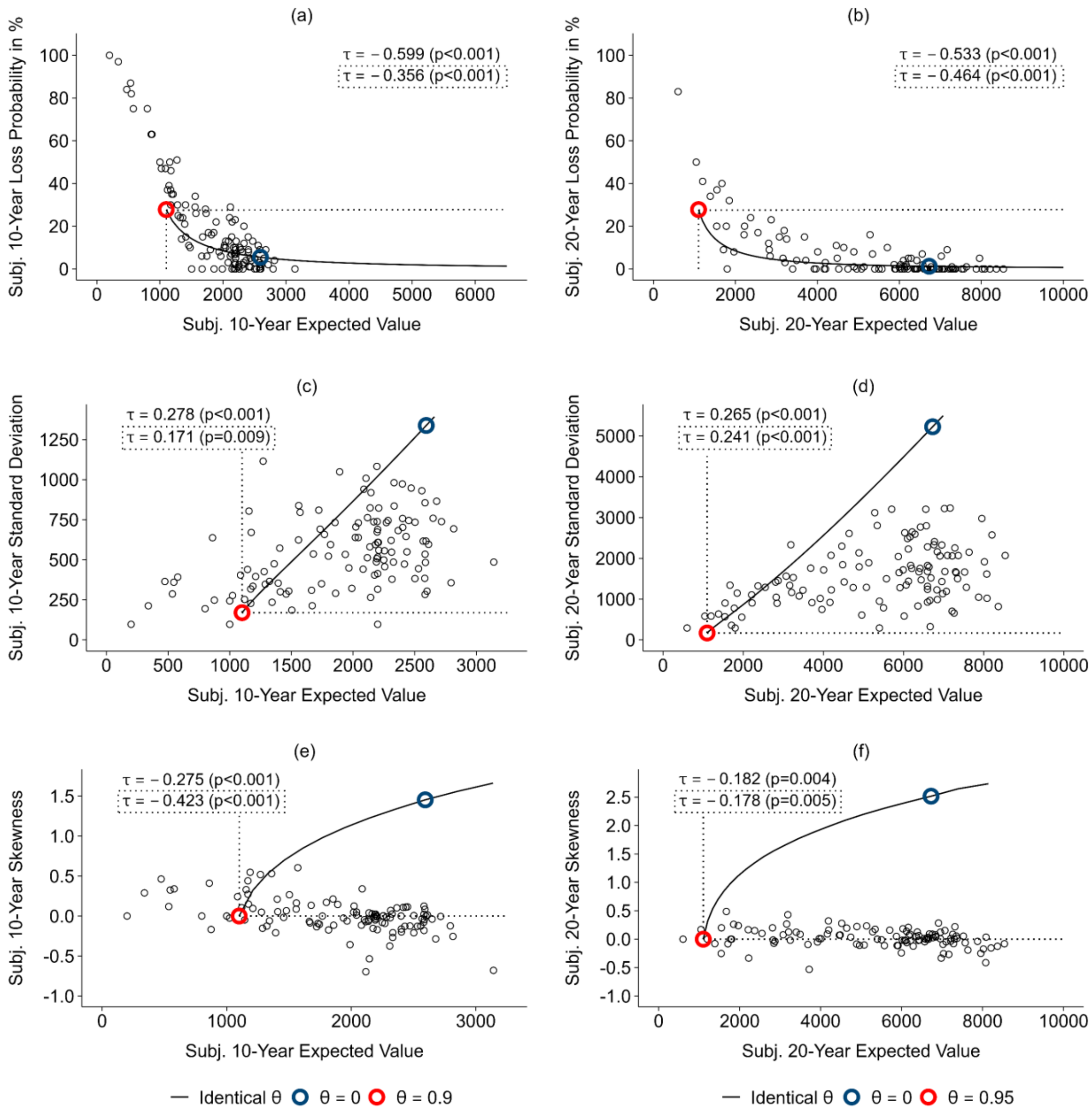


Figure B.1.2: Levels of θ regarding different distributional estimates (Slider Task).



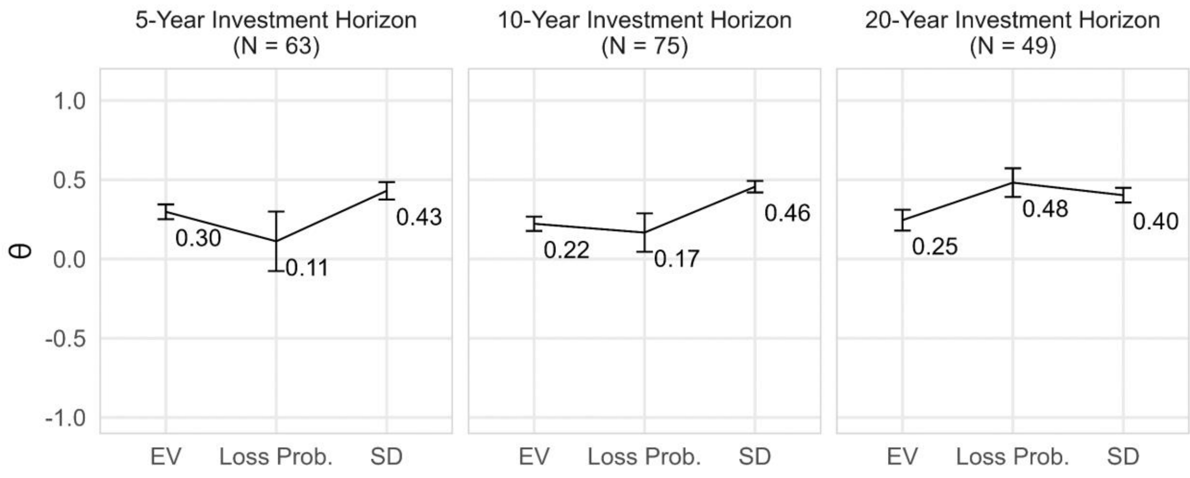
Notes. EV = Expected Value, Loss Prob. = Loss probability, SD = Standard deviation. As mentioned in Chapter 4.1, the range of the slider task is likely to have influenced the subjective expected values in the 20-year condition, potentially reducing bias. Regarding the loss probability, a total of 66 participants estimated a value of 0 in the 20-year condition, removing them from the figure, as this would imply a θ of ∞. A total of 103 participants estimated negative skewness in at least one slider task, which also led to their exclusion from the graph.

Figure B.2.1: Consistency Within Investment Horizons regarding Estimates of the Slider Tasks.



Notes. The left legend applies for panels (a), (c) and (e) and the right legend applies for panels (b), (d) and (f).

Figure B.2.2: Levels of θ regarding different Investment Horizons (Slider Tasks).



Notes. EV= Expected Value, Loss Prob. = Loss probability, SD = Standard deviation. Skewness was omitted for this figure, as 103 participants estimated negative skewness in at least one slider task, which led to their exclusion from the graph. Regarding the loss probability, a total of 66 participants estimated a value of 0 in the 20-year condition, which implies a θ of ∞, removing them from the figure and therefore reducing the sample size to 49.

Appendix C

JAGS code for the robust Bayesian estimation:

```
model {
  for(i in 1:N){
    y[i] ~ dt(mean, 1/(sigma)^2, nu)
  }
  mean ~ dunif(0,1000)
  sigma ~ dunif(0,1000)
  nu       = nuMinusOne + 1
  nuMinusOne ~ dexp(1/29)
}
```

where y[i] refers to the individual estimates of $\Delta_{\theta,10}$ and $\Delta_{\theta,20}$, respectively.

Appendix D

A difference of 5% between the subjective expected value $E\left(FV^{perceived}\right)$ (equation (2)) and $E\left(FV_{dist}^{perceived}\right)$ (equation (6)) is considered practically equivalent. To parameterize the region of practical equivalence (ROPE), the following ratios are set:

$$\frac{PV*(1+i)^{(1-(\theta-x))t}}{PV*(1+i)^{(1-\theta)t}} = 1.05 \quad (8)$$

$$\frac{PV*(1+i)^{(1-(\theta-x))t}}{PV*(1+i)^{(1-\theta)t}} = 0.95 \quad (9)$$

where x defines the width of the ROPE. Solving for x yields $x = \frac{log_{1+i}(1.05)}{t}$ for (8) and $x = \frac{log_{1+i}(0.95)}{t}$ for (9). The ROPEs for the EGB regarding the 10- and 20-year investment are: 10-year ROPE = [-0.054; 0.051], 20-year ROPE = [-0.027; 0.026].

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