# A Moments-Based Approach to Anomaly Forecasts and Statistical Limits to Arbitrage

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ABSTRACT

Adopting a top-down approach to statistical limits to arbitrage, we form an anomaly-percentile distribution of 140 anomalies and estimate the first four moments to forecast mispricing. We interpret the mean and skewness of mispricing as proxies for potential abnormal profits, and the variance and kurtosis as measures for uncertainty and downside risk in mispricing. The empirical analysis shows that the mean and skewness (variance and kurtosis) are positively (negatively) associated with future abnormal returns. The negative effects of variance and kurtosis have intensified in recent decades. The negative relation between variance and kurtosis and realized returns increases with positive sentiment, high market volatility, high liquidity, and positive past returns. Our method is not confined to specific anomaly clusters and substantially improves the performance of anomaly-based portfolios. The results provide insights into the ways that uncertainty and opportunity in arbitrage profits influence market behavior.

## 1. Introduction

Market anomalies refer to situations in which prices deviate from their true values, sometimes for extended periods. The coexistence of anomalies with the widely accepted Efficient Market Hypothesis (Fama, 1970) is primarily reconciled through the framework of the limits to arbitrage (Shleifer & Vishny, 1997): the constraints and limitations that prevent traders from fully exploiting potential mispricing. Recently, Da, Nagel, and Xiu (2024) introduce a theoretical framework known as statistical limits to arbitrage. They argue that the assumption that investors possess full knowledge of the functional form and parameters of the data-generating process does not hold in practice, partly because of the prevalence of high-dimensional environments with many factors. Consequently, investors do not fully exploit arbitrage opportunities, as they aim to mitigate the risk arising from this incomplete knowledge.

In this paper, we adopt a top-down approach to statistical limits to arbitrage, by identifying and quantifying the potential and uncertainty in anomaly forecasts. The statistical limits to arbitrages are embedded in signals derived from large dimensions of anomalies. In our analysis, we use 140 well-known anomalies, rank them by percentiles, and estimate the first four moments of the anomaly-percentile distribution for each stock-month. Our proposed method is analogous to constructing portfolios based on the distribution of price returns. However, we emphasize that in our case the moments refer to the anomaly-percentile distribution as indication of potential mispricing and therefore serves as a future forecast. Accordingly, the association we examine is between these moments and realized future abnormal returns, which arise from arbitrage trading and the correction of mispricing.

Specifically, we posit that the mean and skewness of the anomaly-percentile distribution indicates potential mispricing and may generate future abnormal profits. In contrast, the variance and kurtosis of anomaly forecasts serve as proxies for uncertainty surrounding potential mispricing and arbitrage profits. We interpret these moments of anomaly-percentile distribution as key contributors to limits to arbitrage. All else equal, a higher average of anomaly-percentile distribution serves as a forecast for larger mispricing, which would generate higher future abnormal returns when the arbitrage pricing gap is closed. By similar reasoning, greater right (left) skewness in the anomaly-percentile distribution is a forecast for increased likelihood of larger (smaller) mispricing, which would offer higher (lower) arbitrage returns when the arbitrage mispricing gap is closed. In contrast, greater discrepancies across anomaly-percentile distribution increase the risk associated with exploiting potential arbitrage. This discourages arbitrage trading, allowing the mispricing to persist and maintain the pricing gap. Notably, while previous literature provides evidence that anomaly profitability diminishes over time, we find that the effects of variance and kurtosis have increased over time, consistent with the emergence of large-scale factor models. Such models have become more prevalent in investors' trading decisions because they improve the accuracy of modeling the behavior of asset returns (Didisheim, Ke, Kelly & Malamud, 2024).

Our empirical analysis spans more than 50 years, starting in 1971, when U.S. data for most anomalies first became available. For each stock-month, we calculate 140 anomalies (see Appendix A for details) and assign a percentile rank to each anomaly across all stocks. This process results in a distribution of 140 scores for each stock-month. Next, we compute the four moments of this anomaly-percentile distribution for each stock-month: mean, variance, skewness, and kurtosis. The empirical results support our main hypothesis regarding these moments. First, higher mean and positive skewness in anomaly-percentile distribution are significantly associated with higher future returns. This is because both indicate a higher likelihood of larger mispricing, which translates to higher abnormal returns in the future once the pricing gap is closed. In contrast, higher variance and kurtosis, and more negative skewness in anomaly-percentile distribution, are linked to lower future abnormal returns. These moments reflect increased risks in arbitrage trading, which discourages investors from exploiting mispricing signals.  In other words, when variance and kurtosis are high, arbitrage profits are not realized despite mispricing being indicated by the mean anomaly forecast.

The effects of variance and kurtosis are economically large, with first-month average returns of about −0.46% and −0.82%, respectively. These effects persist over the 6-month future horizon before diminishing. Conditional on market states, we find that the negative relation between variance and kurtosis and future returns becomes stronger with positive sentiment, high market volatility, high liquidity, and positive past returns. These market conditions are often associated with greater anomaly profitability (e.g., Cooper, Gutierrez, and Hameed, 2004; Stambaugh, Yu, and Yuan, 2012; Constantinos, Doukas, and Subrahmanyam, 2013; Wang & Xu, 2015; and Avramov, Cheng, and Hameed, 2016). An exception is skewness, which shows a greater positive impact during periods of negative sentiment, probably because traders rely more heavily on skewness as a supportive signal in less optimistic market conditions.

An important aspect of our proposed statistical limits to arbitrage is that it does not stem from technical or mechanical market characteristics. Rather, its nontechnical nature suggests it is unlikely to disappear in the near term. Chordia, Subrahmanyam, and Tong (2014) show that anomaly returns have declined in recent years, coinciding with reductions in market frictions, such as transaction costs (see also Jacobs and Müller, 2020; Chu, Hirshleifer, and Ma, 2020; and Kaplanski, 2023). Consistent with these findings, we observe a significant weakening in the effects of the mean and skewness of the anomaly-percentile distribution in recent years. In contrast, the variance and kurtosis effects have intensified during the sample period. A similar pattern is observed when the sample is restricted to previously discovered anomalies, following McLean and Pontiff's (2016) definition. While the impact of the mean and skewness of the anomaly-percentile distribution diminishes after discovery, the relative influence of variance and kurtosis increases. This divergence aligns with the notion that limits to arbitrage that arise from uncertainty in anomaly signals do not diminish over time. The likely reason is that as arbitrageurs become more sophisticated, they place greater emphasis on aggregating and integrating information from various sources, including numerous anomaly signals.

Next, we examine whether the information regarding statistical limits to arbitrage is concentrated in a subset of anomalies. We test this prediction in two ways. First, we repeat the main analysis using a subsample of anomalies that have shown profitability in the past, as measured by their past positive alpha. According to Da, Nagel, and Xiu (2024), arbitrageurs statistically learn about anomalies alphas and therefore focus on anomalies with a proven track record of profitability. The results show that the impact of mispricing moments persists within this subset of positive-alpha anomalies, further supporting their relevance in explaining the statistical limits to arbitrage. Moreover, the analysis demonstrates that our proposed source of limits to arbitrage is a distinct case of statistical learning that exists independently of the process of anomaly alpha learning.

In the second test, we analyze the mispricing moments within each of the 13 clusters of anomalies proposed by Jensen, Kelly, and Pedersen (2023) and assess their impact relative to the effects of the all-anomaly moments. Notably, the impact of the moments across anomalies is not confined to specific clusters, although the contribution of within-cluster effects varies substantially across clusters.

Lastly, we perform portfolio analysis based on the information from the anomaly-percentile distribution to construct long, short, and zero-cost (long-minus-short) portfolios. Specifically, stocks are ranked according to the mean of the anomaly-percentile distribution, with adjustments for anomaly forecast variance, kurtosis, and skewness. As customary, the long and short portfolios include stocks within the top and bottom deciles of the rankings, respectively. This analysis reveals that integrating variance, kurtosis, and skewness of anomaly forecasts into the portfolio construction process enhances anomaly investment strategies based on mean forecasts. Penalizing stocks for variance and kurtosis risks while rewarding them for skewness leads to significant improvements in the mean returns, alphas, and Sharpe ratios of portfolios. We provide an illustration of investment performance to demonstrate the impact of the anomaly-percentile distribution moments on returns. The stronger impact of kurtosis suggests that arbitrageurs are more concerned with the risk of negative fat tails in assessing mispricing and arbitrage opportunity than with volatility. In addition, an analysis of those portfolios confirms that the effect of anomaly uncertainty on performance has increased over time.

Our study contributes to the literature on limits to arbitrage by identifying and quantifying a unique source: uncertainty in mispricing signals. De Long, Shleifer, Summers, and Waldmann (1991), and Shleifer and Vishny (1997) emphasize noise trader risk—the risk that mispricing could worsen in the short term—as a key source of limits to arbitrage. Behavioral biases, such as overconfidence (e.g., Daniel, Hirshleifer, and Subrahmanyam, 1998) and loss aversion, are important drivers of noise trader risk (e.g., Barberis & Thaler, 2003; D'Avolio, 2002; Gromb & Vayanos, 2002; Lamont & Thaler, 2003; Jones & Lamont, 2002; Nagel, 2005).[1] Hansen and Sargent (2001) demonstrate that concerns about model misspecification can lead to more conservative trading strategies that fail to fully exploit arbitrage opportunities. Building on this perspective, we examine the effects of uncertainty in mispricing signals on the exploitation of mispricing. By measuring limits to arbitrage using

[1] Additional sources of limits to arbitrage arise from market frictions, which include the costs of identifying mispricing and the resources required to exploit it. Examples of these frictions include transaction costs, such as commissions, bid–ask spreads, and price impact (Pontiff, 1996; Acharya and Pedersen, 2005; Jacoby, Fowler & Gottesman, 2000), as well as holding costs and mark-to-market pressures caused by temporary market trends (Shleifer and Vishny, 1997). Limits to arbitrage also arise from capital constraints (Madhavan and Cheng, 1997), liquidity shocks (Brunnermeier and Pedersen, 2009), and short-selling constraints (Miller, 1977; Chaboud, Chiquoine, Hjalmarsson, and Vega, 2014). Moreover, institutional constraints and regulations (Tufano, 1996) and financial constraints that force arbitrageurs to liquidate positions may contribute to the persistence of mispricing (Gromb and Vayanos, 2002).

statistical inferences drawn from numerous anomaly signals, we quantify risk and reward measures associated with potential arbitrage opportunities. These measures provide valuable insights into the ways that uncertainty in arbitrage opportunities influences investor behavior.

Our study also contributes to the growing body of literature on large-factor models. Studies such as Martin and Nagel (2022), Da, Nagel, and Xiu (2024), Kelly, Malamud, and Zhou (2024), and Didisheim, Ke, Kelly, and Malamud (2024) examine statistical learning in high-dimensional asset pricing models. They provide a theoretical foundation for statistical learning, which is typically rooted in machine learning methodologies. We employ a related perspective, taking a top-down approach to the subject, enabling us to address the black-box nature of such models. Rather than utilizing all the data to infer statistical patterns, sometimes without a priori motivation, we focus on statistics formed by prior knowledge about their expected impact on arbitrage trading. This approach allows us to directly test the economic reasoning underlying our empirical analysis while offering deeper insights into how statistical learning influences market dynamics.

An additional contribution of our study relates to anomalies in general and their association with statistical limits to arbitrage. For instance, Jensen, Kelly, and Pedersen (2023) provide compelling evidence of numerous anomalies across 93 countries that consistently yield excess returns on risk. They also demonstrate that some anomalies have strengthened rather than weakened over time. Moreover, the improvement in market efficiency over the past decades shows that the information in many market anomalies is, at least partly, priced (e.g., Chordia, Subrahmanyam and Tong, 2014). However, uncertainty regarding anomaly signals remains a source of limits to arbitrage. Our findings show that while market efficiency increases over time, there are still gaps in the process of information inflow and its incorporation in prices.

## 2. Data and methodology

The empirical analysis includes all U.S. firms listed on the NYSE, AMEX, and NASDAQ with share codes 10 and 11 from the CRSP database. The sample includes returns of delisting stocks. If delisting return data are missing and the delisting is performance-based, we follow Shumway (1997) by setting the delisting return to −30%. To mitigate backfilling biases (Fama and French, 1993), a firm must have been listed on Compustat for at least two years to be included in the sample. Microcap stocks that fall below the NYSE benchmark for the bottom 20% of stocks by market capitalization are excluded from the sample. This exclusion reduces microcap biases and eliminates stocks less relevant for trading due to severe trading frictions, significant liquidity issues, or lack of information.

The financial statement data are from the Compustat database. The figures for the previous fiscal year are updated annually at the end of June, ensuring that real-time information is available for predicting future stock returns. Quarterly financial statements are updated monthly, provided the release date is known. If the release date is not specified, the data are assumed to be public by the end of the fourth month following the reporting period. As many anomalies require several years of historical data, we use data dating back to 1960. However, due to the absence of many Compustat variables in the early years, our empirical analysis starts in 1971, when data for most anomalies are available, and ends in 2022. In total, we have 1,601,774 firm-month observations across 13,039 firms.

We explore 140 anomalies (see Appendix A) proposed in the literature as being correlated with future stock returns. In calculating anomalies, we adhere strictly to the methodological guidelines of Jensen, Kelly, and Pedersen (2023), who offer a comprehensive framework for their computation. The calculation is as follows: first, we calculate the anomalies each stock-month. Then, we assign a percentile rank to each anomaly across all stocks, obtaining 140 percentile ranks across all stocks each month. We denote the percentile rank for each anomaly $i$ for stock $j$ in month $t$ as $x_{i,j,t}$. This standardization using percentage ranking facilitates comparisons across anomalies and reduces the impact of outliers. In addition, it allows us to align the direction of all anomalies by taking one minus the percentile rank $(1 - x_{i,j,t})$ for anomalies known to be negatively correlated with future returns.

Next, for each stock-month observation, we calculate the first four moments of the anomaly distribution to measure stock mispricing and potential arbitrage opportunities. As mispricing is expected to be positively associated with future abnormal returns, we interpret the four moments of mispricing as proxies for the corresponding moments of future abnormal returns. The first moment of stock price distribution represents the expected abnormal return on that stock. Similarly, we hypothesize that the first moment of the anomaly forecast distribution represents the average aggregate mispricing across all anomalies. Denoted as $E(x_i)_{j,t}$, it represents a stock's relative strength in generating future abnormal returns. This mispricing is expected to be positively correlated with future abnormal returns on that stock when the mispricing is corrected. Thus, a higher mean forecast indicates a greater likelihood of higher future abnormal returns.

Continuing with this analogy, just as the variance of a stock's price returns reflects the volatility of expected future returns around the mean, the variance of a stock's anomaly forecasts, $\sigma^2(x_i)_{j,t}$, represents the uncertainty in mispricing surrounding the mean of expected abnormal returns. For risk-averse arbitrageurs, we hypothesize that, all else being equal, a higher variance in anomaly forecasts leads to a discount on potential abnormal returns, as volatility discourages the exploitation of arbitrage opportunities. This reduced willingness to exploit arbitrage opportunities is expected to result in persistent mispricing and an absence of realized abnormal returns in the near future. As a result, we anticipate that stocks with higher anomaly forecast volatility will yield lower future abnormal returns compared to stocks with the same mean forecast but lower volatility.

Similarly, the fourth moment (kurtosis), denoted as $Kurt(x_i)_{j,t}$, indicates fatter tails and higher downside risk in anomaly forecasts and implied mispricing. If arbitrageurs are averse to downside risk, they are likely to discount potential mispricing with higher tail risk, as measured by kurtosis. We hypothesize that greater downside risk discourages investors from fully exploiting mispricing, producing an effect similar to that of volatility. Consequently, we expect lower future realized returns for stocks with higher kurtosis compared with stocks that have the same average forecasts but lower kurtosis.

To complete the analysis, we also examine the skewness of the distribution of forecasts, $Skew(x_i)_{j,t}$. Harvey and Siddique (2000) show that, all else being equal, investors prefer right-skewed portfolios to left-skewed ones. Analogously, we hypothesize that positive skewness of anomaly distribution increases the likelihood of large mispricing. This is because positive skewness implies a longer right tail in anomaly forecast distribution. As arbitrageurs are attracted to higher likelihood of large mispricing, they may inflate their expectations of obtaining abnormal returns in response to greater positive skewness. This, in turn, encourages trading on potential mispricing, thereby increasing future realized abnormal returns. Conversely, if the anomaly distribution is left-skewed (i.e., a negative skewness), it implies a longer tail on the left, increasing the risk associated with exploiting arbitrage opportunities and imposing limit on arbitrage.

We analyze two sample sets. In the main analysis, we use the four moments on all 140 anomalies. In a separate analysis, we consider anomaly clusters classified according to Jensen, Kelly, and Pedersen (2023) (see Appendix A). For each month-stock, in addition to the four moments obtained from all anomalies, we also compute the four moments from anomalies belonging to each cluster separately. This approach allows us to distinguish the distinct contribution of anomalies within each cluster from the overall effect.

Table 1 presents descriptive statistics for the four moments of the distribution of anomalies' forecasts for the entire sample period. The average anomaly forecast is 0.5, resulting from the percentile ranking procedure. The standard deviation is 0.27, the skewness is 0.03, and the kurtosis is 1.94. For the analysis of the anomaly clusters, we present the four moments for each cluster. The average forecast for each cluster is also close to 0.5. However, the other moments vary across the clusters. The average standard deviation ranges from 0.13 for momentum anomalies to 0.26 for seasonality anomalies. The skewness also varies significantly, ranging from −0.06 for low leverage to 0.12 for quality anomalies, and the kurtosis ranges from 1.73 for skewness to 3.21 for low-risk anomalies.

## 3. Cross-sectional regressions

In this section, we employ the Fama and MacBeth (1973) cross-sectional regression framework. Each month, we regress stock returns on the mean, variance, skewness, and kurtosis of anomaly forecasts. Formally, the monthly regression equation at time *t* is:

$$R_{j,t+1:t+m} = \alpha_t + \beta_{1t}E(x_i)_{j,t} + \beta_{2t}\sigma^2(x_i)_{j,t} + \beta_{3t}Skew(x_i)_{j,t} + \beta_{4t}Kurt(x_i)_{j,t} + Controls_{j,t} + \varepsilon_{j,t}, \quad (1)$$

where $R_{j,t+1:t+m}$ is the return on stock *j* from time $t + 1$ to time $t + m$; *m* represents 1, 3, 6 or 12 months; $E(x_i)_{j,t}$ is the average of 140 anomalies' percentile-ranked forecasts of firm *j* at time *t*; and $\sigma^2(x_i)_{j,t}$, $Skew(x_i)_{j,t}$, and $Kurt(x_i)_{j,t}$ are their variance, skewness and kurtosis, respectively. The firm's control variables include the log of market equity value, the book-to-market ratio for firms with a positive book value (0 otherwise), idiosyncratic risk measured as the 60-month volatility of residuals from the CAPM model, and the average turnover (traded shares scaled by total number of shares) over the past 126 trading days.

Table 2 presents the slope coefficients of the regressions. The dependent variable in the first column is the next month's return. The mean of anomaly forecasts has a highly significant slope coefficient of 0.0551 ($t$-statistic = 6.67), indicating that the stock's average anomaly strongly predicts next month's return. Since the mean percentile-ranked is 0.5 (see Table 1), this coefficient implies an average monthly return forecast on anomalies of $0.0551 \times 0.5 = 0.0275$ or about 2.75%.[2] The variance and kurtosis slope coefficients are −0.0625 and −0.0042, both highly significant ($t$-statistic = −3.22 and −5.00, respectively). These results support our main hypothesis that higher arbitrage risk, as implied from the volatility and fat tails of the anomaly forecast distribution, is associated with lower future realized abnormal returns. Considering the mean of the variance and kurtosis to be $0.27^2 = 0.0729$ and 0.031, respectively, the size of the coefficients implies a decrease in the average monthly return forecast of $-0.0625 \times 0.0729 = -0.0046$, or −0.46%, for the variance and $-0.0042 \times 1.94 = -0.0082$, or −0.82%, for the kurtosis of anomalies. The skewness slope coefficient of 0.0040 is also significant ($t$-statistic = 3.10), suggesting that positive skewness in anomaly forecasts is followed by higher future realized returns. This finding aligns with the hypothesis that positive skewness encourages the exploitation of arbitrage which, in turn, inflates future realized abnormal returns. The size of the coefficient implies an average monthly return forecast of $0.004 \times 0.003 = 0.00012$, or 0.012%, due to skewness in anomalies' distribution.

The dependent variable in the regressions reported in the other columns is the stock's

[2] Note that this large value is based on the mean of all the firms without accounting for their size (apart from the preliminary elimination of the 20% smallest firms). In the portfolio analysis, we adhere to value-weighted portfolios to also account for firm size.

future returns over 3, 6, and 12 months. The results are consistent with those for next month's returns and support our main hypotheses. The coefficients initially increase in absolute terms with the horizon and then decline for the 12-month returns. The mean slope coefficient rises to 0.1383 for the 3-month returns and 0.2482 for the 6-month returns, before decreasing to 0.1523 for the 12-month returns (*t*-statistic = 6.62, 7.06, and 4.57, respectively). The skewness slope coefficients follow a similar pattern at 0.0092, 0.0165, and 0.001 (*t*-statistic = 2.77, 2.88, and 0.03, respectively). The variance slope coefficients are –0.1920, –0.3301, and –0.2882 (*t*-statistic = –3.77, –3.96, and –2.37), whereas the kurtosis slope coefficients are –0.0118, –0.0208, and –0.0162 (*t*-statistic = –5.41, –5.37, and –4.68) for the 3-, 6-, and 12-month returns, respectively

To sum up, according to Table 2, the mean and skewness are positively associated with future realized returns, while the variance and kurtosis are negatively associated. The effects of the variance and kurtosis are economically large, at an average monthly rate of about –0.5% and –0.82%, respectively. The positive correlation of the mean is expected, given that anomalies are, by definition, correlated with abnormal returns. The positive association of skewness suggests that a higher potential for future abnormal returns encourages arbitrage trading. In contrast, higher variance and kurtosis indicate that higher arbitrage risks discourage arbitrage trading.

### 3.1 Limit to arbitrage over time

Trading on arbitrage opportunities has substantially expanded over time because of declining market frictions, such as transaction costs, and the increased flow of information, as suggested by Chordia, Subrahmanyam, and Tong (2014) and further demonstrated by Jacobs and Müller (2020), Chu, Hirshleifer, and Ma (2020), and Kaplanski (2023). Building on this evidence, we argue that the growing intensity and sophistication of arbitrage trading involves a broader set of anomalies and greater reliance on comparing and integrating their underlying information. Consequently, discrepancies across anomalies are expected to play an increasingly important role in traders' decision-making and the underlying realized returns, despite the general decline in abnormal returns over time. To confirm this prediction, we first compare the early and more recent years, and then perform a second analysis that each period relate only to anomalies that previously identified, as suggested by McLean and Pontiff (2016). In Panel A of Table 3, the sample period is divided into two: an earlier period, 1971–1999, on the left side of the table; and a recent period, 2000–2022, on the right sides of the table. For each period, we run Fama–MacBeth regressions, similar to the analysis in Table 2. Consistent with the findings from prior studies regarding the increase in arbitrage trading over the years, the coefficients for the mean anomaly forecast exhibit a noticeable decline across all future horizons. For the 1971–1999 period, the slope coefficients for the 1-, 3-, 6-, and 12-month returns are 0.0775, 0.1953, 0.3607, and 0.1814, respectively, all of which are highly significant (*t*-statistic = 8.02, 8.14, 9.17, and 4.45). In contrast, the coefficients for the years 2000–2022 are substantially smaller, with coefficient estimates of 0.0265, 0.0653, 0.1026, and 0.1132, for the 1-, 3-, 6-, and 12-month returns, respectively, and lower significance levels (*t*-statistic = 1.97, 1.76, 1.76, and 2.04, respectively). These findings indicate that the profitability of anomaly forecasts has weakened over time, aligning with the hypothesis that the increasing market efficiency due to more arbitrage activity and lower market frictions has compressed abnormal returns over time.

The skewness coefficients in the third line reflects a similar decline. For the 1971–1999 period, the slope coefficients are 0.0080, 0.0193, 0.0345, and 0.0074, for the 1-, 3-, 6-, and 12-month returns, respectively, and the first three coefficients are highly significant (*t*-statistic = 4.95, 4.43, 4.69, and 1.06, respectively). However, during the 2000–2022 period, the coefficients became insignificant. This shift suggests that the influence of skewness on future realized returns has diminished in the recent period.

In sharp contrast, the variance and kurtosis coefficients became more negative with higher significance levels in recent years. For 1971–1999, the variance coefficients are –0.0401, –0.1294, –0.2385, and –0.1795 for the 1-, 3-, 6-, and 12-month returns, respectively (*t*-statistic = –1.98, –2.55, –2.71, and –1.84, respectively), but for 2000–2022, these coefficients are considerably more negative, with values of –0.0910, –0.2720, –0.4485, and –0.4344, respectively (*t*-statistic = –2.54, –2.87, –2.98, and –2.90, respectively). Similarly, for 1971–1999, the kurtosis coefficients for the 1-, 3-, 6-, and 12-month returns, are –0.0025, –0.0091, –0.0180, and –0.0122, respectively (*t*-statistic = –2.95, –4.05, –3.86, and –3.05, respectively), and shift to –0.0065, –0.152, –0.0239, and –0.0215 (*t*-statistic = –4.17, –3.96, –3.88, and –3.60, respectively) for 2000–2022.

In Panel B of Table 3, the sample is limited to previously discovered anomalies. To estimate the timing of discovery, we adopt the definition of McLean and Pontiff (2016), and include an anomaly in our sample starting from the end of the sample period of the first study that revealed this anomaly. The underlying assumption is that, once an anomaly is identified, information about it becomes publicly available and influences the market.

The results of the analysis are in line with the results in Panel A and the existing literature. The coefficient for the mean anomaly forecast for next month's return of 0.0267 (*t*-statistic = 3.40) is nearly half the size of the corresponding coefficient when using the entire sample period (as in Table 2). The coefficients for longer horizons remain substantial and positive but are no longer significant, suggesting that arbitrage profits not only diminish after the discovery of anomalies but also materialize more quickly. Additionally, in alignment with the results in Panel A, the skewness coefficients become statistically insignificant.

In sharp contrast, the variance and kurtosis coefficients remain highly significantly negative at −0.0230, −0.0303, −0.0262, and −0.1068 ($t$-statistics = −2.29, −3.06, −2.77, and −2.30), and −0.0029, −0.0031, −0.0029, and −0.0137 ($t$-statistics = −3.98, −4.05, −4.08, and −4.64), respectively. Given the decline in arbitrage profits, as reflected in the reduced coefficient of the mean anomaly forecast, the relative impact of variance and kurtosis appears substantially more pronounced among discovered anomalies.

To sum up, similar to the results in Table 2, the findings in Table 3 also show an increasingly negative effect of statistical limits to arbitrage, as implied by discrepancies across anomalies on future realized returns. This negative effect is more profound in recent years and among discovered anomalies. Given the overall decline in mean future returns over time, this suggests that variance and kurtosis exert a relatively stronger impact on diminishing realized returns.

### 3.2 Limit to arbitrage under varying market conditions

In this section, we examine the impact of market conditions on our proposed limits to arbitrage. Specifically, we divide the sample according to high and low investor sentiment, volatility, liquidity, and market returns, and explore the impact of the distribution of anomaly forecasts conditional on these economic states. We conjecture that the impact of our proposed limits to arbitrage measures would be more pronounced when market conditions align with more profitable anomalies.

Panel A of Table 4 presents regression results similar to those in Table 2 with the sample divided into months of negative and positive sentiment, as defined by the Baker and Wurgler

(2006) sentiment index.[3] The literature shows that the returns on key anomalies are higher during periods of higher sentiment. For instance, Stambaugh, Yu, and Yuan (2012) and Constantinos, Doukas, and Subrahmanyam (2013) demonstrate that momentum profitability increases during high sentiment periods. Consistent with these studies, our results show that the mean slope coefficients are substantially larger when the sentiment is positive. Specifically, for the 1-, 3-, 6-, and 12-month returns, the coefficients are 0.1523, 0.1673, 0.2862, and 0.1621, respectively ($t$-statistic = 4.57, 5.60, 5.73, and 3.20), compared with 0.0475, 0.1013, 0.1985, and 0.1412 ($t$-statistic = 4.23, 3.82, 4.33, and 3.56) when the sentiment is negative.

The negative impact of variance and kurtosis also becomes more pronounced with positive sentiment. Under negative sentiment, the variance coefficients of −0.0227, −0.0717, −0.1622, and −0.2888 for the 1-, 3-, 6-, and 12-month returns, respectively, are mostly insignificant ($t$-statistic = −0.97, −1.21, −1.56, and −2.61, respectively),). However, under positive sentiment, the regression coefficients for the 1-, 3-, 6-, and 12-month returns become highly significant at −0.2888, −0.3137, −0.5057, and −0.2879, respectively ($t$-statistic = −3.37, −4.13, −4.30, and −2.39, respectively). A similar trend is observed in the kurtosis coefficients, which shift from −0.0018, −0.0064, −0.0138, and −0.0177 ($t$-statistic = −1.82, −2.45, −2.64, and −3.91, respectively) for the 1-, 3-, 6-, and 12-month returns, respectively, under negative sentiment to −0.0162, −0.0163, −0.0265, and −0.0141 ($t$-statistic = −4.68, −5.35, −5.52, and −2.87, respectively) under positive sentiment. The more negative coefficients during the positive sentiment period indicate that arbitrage risk has a stronger negative effect on future realized returns.

Lastly, the skewness coefficients show a diminishing influence with rising sentiment.

[3] Baker and Wurgler's (2006) equity market sentiment that captures variations in six different time series measures that proxy investor sentiment. These measures are the discount on close-end funds, turnover, the number of IPOs, average first-day returns, the equity share of new issues, and dividend premium.

For the 1-, 3-, 6-, and 12-month returns, these decrease from 0.0054, 0.0124, 0.0277, and 0.0154, respectively ($t$-statistic = 2.99, 2.67, 3.36, and 2.08, respectively) when sentiment is negative to 0.0001, 0.0052, 0.0052, and −0.0135, respectively ($t$-statistic = 0.03, 1.17, 0.72, and −1.62, respectively) when sentiment is positive.

Panel B of Table 4 presents the impact of market volatility. We divide the sample into two subperiods, categorized above and below the historical median of monthly volatility, calculated from the daily returns of the total market value-weighted index. For each month $t$, we calculate the median of historical monthly volatilities from the beginning of the sample period until month $t - 1$ and categorize month $t$ accordingly. Wang and Xu (2015) point out that high market volatility is linked to a decline in momentum payoffs. However, we find that the slope coefficients for the means are notably larger and more significant during low volatility periods. For these periods, the coefficients are 0.0594, 0.1525, 0.2825, and 0.1828 for the 1-, 3-, 6-, and 12-month returns, respectively ($t$-statistic = 6.07, 6.88, 7.63, and 4.17, respectively) compared with 0.0516, 0.1258, 0.2180, and 0.1248, respectively ($t$-statistic = 3.95, 4.09, 4.16, and 2.85, respectively) for high volatility periods.

The skewness coefficients demonstrate a similar trend. When volatility is low, the skewness coefficients are 0.0052, 0.0104, 0.0211, and 0.0077 for the 1-, 3-, 6-, and 12-month returns, respectively ($t$-statistic = 3.19, 2.88, 3.22, and 1.08, respectively), but when volatility is high, they decrease to 0.0028, 0.0081, 0.0124, and −0.0066, respectively, and become insignificant ($t$-statistic = 1.53, 1.79, 1.54, and −0.84, respectively). This finding implies that traders' reliance on skewness reduces when market conditions are less stable.

The findings for variance and kurtosis of anomalies are reversed. Their negative impact becomes more pronounced during periods of high market volatility. For the variance, the coefficients shift from −0.0227, −0.0717, −0.1622, and −0.2888 for the 1-, 3-, 6-, and 12-month returns, respectively ($t$-statistic = −0.97, −1.21, −1.56, and −2.61, respectively) during low volatility periods to −0.2888, −0.3137, −0.5057, and −0.2879, respectively ($t$-statistic = −3.37, −4.13, −4.30, and −2.39, respectively) during high volatility periods. This indicates that arbitrage risk, as measured by variance, has a more significant negative impact on future realized returns when market volatility is high, highlighting the increased sensitivity of anomaly realized returns to risk in turbulent market conditions.

A similar trend is observed with kurtosis. The coefficients during low volatility periods are −0.0018, −0.0064, −0.0138, and −0.0177 for the 1-, 3-, 6-, and 12-month returns, respectively ($t$-statistic = −1.82, −2.45, −2.64, and −3.91, respectively), and shift to −0.0162, −0.0163, −0.0265, and −0.0141, respectively ($t$-statistic = −4.68, −5.35, −5.52, and −2.87, respectively) during high volatility periods. The increased negative influence of kurtosis indicates that higher-order moments of arbitrage risk are particularly detrimental in volatile markets, as they likely amplify the uncertainty arbitrageurs face.

Panel C presents the estimation results for different levels of market liquidity. Market liquidity is stratified by the historical median of the monthly Amihud (2002) measure. Each month $t$, we calculate the median historical of a monthly Amihud measure from the beginning of the sample period until month $t-1$, and categorize month $t$ accordingly. Avramov, Cheng, and Hameed (2016) find that momentum is stronger in highly liquid markets. In our setting, we find that the positive slope coefficients for mean and skewness and the negative coefficients for variance and kurtosis are mostly insignificant during high illiquidity periods but become significant when illiquidity is low. This finding indicates that the positive effects of mean and skewness, and the negative effects of variance and kurtosis, are substantially more pronounced in liquid markets. The intuition is that when market liquidity diminishes, arbitrage trading is scarce and realized abnormal returns are minimal and difficult to notice.

The same phenomenon is observed in Panel D, which presents results for periods of positive and negative market returns based on the past 2-year market performance, as defined by Cooper, Gutierrez, and Hameed (2004). Here, the slope coefficients, which are largely insignificant during periods of negative past returns, become highly significant during periods of positive past returns. Thus, the positive effects of mean and skewness, along with the negative effects of variance and kurtosis, are concentrated in periods following positive market returns.

To sum up, the association between anomaly forecasts and future realized returns highly depends on market conditions. This dependency is stable across investment horizons ranging from a month to a year. The correlation between forecasts' means and future realized returns is stronger in the early years of the sample. It is also higher during periods of positive sentiment, low volatility, high liquidity, and positive past returns. These findings align with the empirical evidence in the literature, particularly regarding well-known anomalies such as momentum.

Skewness follows a similar pattern, with a notable exception: it shows a higher positive impact when sentiment is negative. Thus, the positive correlation with future returns becomes particularly significant during periods of low sentiment. Plausibly, traders place more emphasis on skewness as a supportive signal in less optimistic market conditions.

Variance and kurtosis exhibit a similar pattern in their effects on future realized returns. They are significantly negative under most conditions and periods, except when markets are illiquid. However, the main takeaway is that their negative impact has been more pronounced in recent years, aligning with the evidence for intensified sophisticated arbitrage trading and lower realized returns on anomalies. Moreover, their negative effects are more substantial when sentiment is positive, volatility is high, markets are liquid, and past returns are positive. Altogether, their influence intensifies when the association between anomaly means and realized returns is stronger.

### 3.3 Limit to arbitrage within and across anomaly clusters

Jensen, Kelly, and Pedersen (2023) argue that anomalies can be clustered into a small number of clusters of highly correlated anomalies. They propose a classification of 13 clusters that possess a high degree of within-return correlation and economic concept similarity, while the degree of correlation across the clusters is low. In this section, we examine whether the within-return correlation in anomaly clusters adds to the information content of the four moments on all anomalies. A specific interest is whether the arbitrage risk implied by volatility and kurtosis stems from uncertainty within anomaly clusters. That is, do investors decrease their arbitrage trading when there is higher discrepancy among similar anomalies within the same cluster?

To address this research question, we regress Equation (1) for each cluster separately. That is, for each cluster *k*, we assign a percentile rank to the anomalies in the cluster, calculate the four moments for each cluster, and use these moments as additional explanatory variables for future returns. Formally, the regression is as follows:

$$R_{j,t+1:t+m} = \alpha_t + \beta_{1t}E(x_i)_{j,t} + \beta_{2t}\sigma^2(x_i)_{jt} + \beta_{3t}Skew(x_i)_{j,t} + \beta_{4t}Kurt(x_i)_{j,t} + +\beta_{5t}E(x_{cl})_{j,t} + \beta_{6t}\sigma^2(x_{cl})_{j,t} + \beta_{7t}Skew(x_{cl})_{j,t} + \beta_{8t}Kurt(x_{cl})_{j,t} + Controls_{j,t} + \varepsilon_{j,t}, \qquad (2)$$

where $R_{j,t+1:t+m}$ is the return on firm *j* from time $t + 1$ to time $t + m$; *m* represents 1, 3, 6, or 12 months; $E(x_i)_{j,t}$, $\sigma^2(x_i)$, $Skew(x_i)$, and $Kurt(x_i)$ are the distribution moments of the forecasts of all 140 anomalies for firm *j* at time *t* as in Equation (1), and $E(x_{cl})_{j,t}$, $\sigma^2(x_{cl})_{j,t}$ $Skew(x_{cl})_{j,t}$, and $Kurt(x_{cl})_{j,t}$ are the distribution moments of the within-cluster anomaly of firm *j* at time *t*. The control variables are the same as in Equation (1).

Table 5 presents the slope coefficients from the regressions in Equation (2) when realized returns are for the next month. The results of 3-, 6-, and 12-month returns are presented in Appendix B. The first four columns correspond to all 140 anomalies. Consistent with previous findings, the slope coefficients for the moments of all 140 anomalies are significantly positive for mean and skewness. For variance and kurtosis, the coefficients are negative and most of them are significant.

Columns (4) – (8) reflect the moments of anomalies within clusters. The slope coefficients for the mean are insignificant except for positive skewness and negative seasonality coefficients, indicating that no single cluster significantly dominates the information from all the anomalies in forecasting future returns. The coefficients for variance are significantly positive for investment, low leverage, profit growth, profitability, seasonality, and value, but are significantly negative for low risk and skewness, which indicates that the negative effect of volatility and skewness on realized returns is particularly strong within these cluster. The coefficients for skewness are largely insignificant. The coefficients for kurtosis are significantly negative for debt issuance, low risk, quality, and skewness, but significantly positive for low leverage, momentum, profit growth, profitability, seasonality, and value. This highlights that the effect of kurtosis within clusters varies substantially across clusters.

The results in Table 5 show that the impact of moments across anomalies is not limited to specific clusters. All the moment effects are more pronounced across the entire sample than within clusters. Moreover, the contribution of within-cluster variance and kurtosis effects varies substantially across clusters. Last, the results with realized returns for longer horizons in Appendix B are similar, except that the contribution of specific clusters is generally more pronounced.

### 3.4 Limit to arbitrage in positive-alpha anomalies

We repeat the analysis, focusing only on anomalies that previously yielded abnormal profits. This approach aligns with the investor statistical learning process proposed by Da, Nagel, and Xiu (2024). We use this approach to test the robustness of our results, assuming that investors prioritize anomalies with a proven track record of profitability. Furthermore, this approach demonstrates that our proposed source of limits to arbitrage represents a distinct form of statistical learning that persists even after accounting for the process of anomaly alpha learning. To identify the set of profitable anomalies at month $t$, we follow this procedure: for each month-anomaly, we construct a zero-cost, decile-based spread portfolio, which consists of equal amounts of long and short value-weighted portfolios. The long portfolio includes stocks in the most extreme decile, which predict positive future returns, while the short portfolio includes stocks predicting negative future returns. Then, we regress the future returns of the spread portfolio on the Fama–French (2015) five factors. An anomaly is included in the four-moment calculations only if its alpha at time $t-1$ is positive, thus focusing the analysis on positive-alpha anomalies that have already demonstrated abnormal returns.

Table 6 presents the slope coefficients of the regressions in Equation (1), where the moments are calculated from the percentile ranks of positive-alpha anomalies. The results are similar to the analysis with all anomalies, as shown in Table 2. The mean and skewness slope coefficients are positive and significant for horizons up to 6 months, while the variance and kurtosis slope coefficients are significantly negative for all horizons. Overall, the conclusions remain unchanged when we confine the analysis to positive-alpha anomalies. Thus, our proposed source of limits to arbitrage is a separate case of statistical learning that exists **independently** of the process of anomaly alpha learning shown by Da, Nagel, and Xiu (2024).

## 4. Portfolio analysis

In this section, we use portfolio analysis to investigate strategies to exploit uncertainty in anomaly forecasts. As customary in cross-sectional portfolio analysis, the investment strategy is based on calculating an all-anomaly percentile-rank forecast, in which the exact calculation method varies depending on the aspect of the anomaly distribution being explored. The investment strategy is performed as follows: First, we define the anomaly forecast method, denoted as $AF_{j,t}(x_i)$, and calculate its value for each month-stock. Then, we sort all stocks each month according to $AF_{j,t}(x_i)$, and form long and short portfolios by selecting stocks from the top and bottom deciles. We denote these portfolios as TOP_$AF_t$ and BOTTOM_$AF_t$, respectively. The stocks in each portfolio are value-weighted according to their market value, and an equal amount is invested in both the long and short portfolios to create a zero-cost portfolio. The return for the next month on this zero-cost portfolio represents the realized return on the all-anomaly forecast portfolio. In each of the following analyses, we report the performance across the entire sample period on the long, short, and zero-cost spread (long-minus-short) portfolios.

We begin by exploring the impact of the variance across anomalies on realized returns. In this respect, we use the percentile-rank monthly means of all 140 anomalies per month-stock, $E(x_i)_{j,t}$ and the percentile-rank variance across anomalies, $\sigma^2(x_i)_{j,t}$, as the features of the anomaly distribution. Assuming arbitrage traders are risk-averse to forecast volatility, they discount potential abnormal profits in response to higher volatility, depending on their degree of volatility aversion. This, in turn, reduces their arbitrage trading and the corresponding realized abnormal returns. Therefore, we investigate the existence of this discounting factor by penalizing the stock's mean forecast for volatility.

Formally, let $A$ represent a volatility-aversion coefficient, which reflects the market degree of forecast's volatility aversion. The all-anomaly forecast per stock-month with volatility discounting is defined as

$$AF(x_i)_{j,t} = \mathrm{E}(x_i)_{j,t} - A \times \sigma^2(x_i)_{j,t} \tag{3}$$

For each month, we sort the stocks according to $AF(x_i)_{j,t}$ and construct long and short portfolios consisting of stocks in the top and bottom deciles, respectively. By changing the volatility-aversion coefficient, we assess how volatility aversion affects the value of anomaly forecasts and their associated realized returns.

Figure 1 presents the performance of an all-anomaly portfolio, based on Equation (3). It plots investment characteristics across the entire sample period (1971 to 2022) for the following portfolios: a long position in the top decile portfolio TOP_AF$_t$, a short position in the bottom decile portfolio BOTTOM_AF$_t$, and the zero-cost spread portfolio. The figure plots these estimates for different values of the volatility-aversion coefficient $A$, ranging from 0 (indicating no penalty for volatility) to 0.8. Panel A displays the next month's average returns across the entire sample period for the three portfolios. The long portfolio's mean return of 1.13% for $A = 0$ (no penalty) peaks at 1.20% for $A = 0.44$ and then decreases with a higher volatility-aversion coefficient. Conversely, the short portfolio shows an opposite pattern, with a mean return of 0.30% for $A = 0$, reaching a minimum of 0.23% at $A = 0.42$ before increasing again. Consequently, the mean return on the zero-cost spread portfolio exhibits a distinct local maximum, rising from 0.83% for $A = 0$ to 0.96% for $A = 0.42$ and then declining.

Panel B presents the alphas obtained from regressing the portfolios' monthly returns on the Fama–French (2015) five factors. The long portfolio's alpha peaks at $A = 0.44$, while the short portfolio's alpha reaches a trough at $A = 0.21$. The spread portfolio alpha increases from 0.33 at $A = 0$ to 0.42 at $A = 0.39$, and then declines. Panel C shows the Sharpe ratios of the portfolios, which follow similar patterns. The Sharpe ratio for the long portfolio—calculated as the average monthly return minus the contemporaneous risk-free rate, scaled by the standard deviation of monthly returns—peaks at 1.35 for $A = 0.44$. The short portfolio's Sharpe ratio, which is calculated in the same way, hits a low at $A = 0.27$. The spread portfolio's Sharpe ratio—calculated as the average monthly return scaled by the standard deviation of monthly returns—increases from 0.74 for $A = 0$ to 0.76 for $A = 0.42$, and then declines.

Figure 1 demonstrates that penalizing stocks for the high volatility of anomalies' forecasts improves mean returns, the alpha corresponding to the Fama–French (2015) five factors, and the Sharpe ratio. The performance of the spread portfolio improves with the increase of the volatility-aversion coefficient up to 0.42 and declines beyond that point.

Next, to examine the presence of a discount factor for kurtosis, we define an all-anomaly forecast that incorporates kurtosis discounting as follows:

$$AF(x_i)_{j,t} = \mathrm{E}(x_i)_{j,t} - A \times Kurt(x_i)_{j,t} \tag{4}$$

Figure 2 displays performance measures of the all-anomaly portfolio, where the mean forecast is penalized for kurtosis as described in Equation (4). The long portfolio's mean return in Panel A peaks at 1.41% ($A = 1.06$), while the short portfolio bottoms at 0.27% ($A = 0.08$). The zero-cost spread portfolio's mean return rises from 0.83% at $A = 0$ to 1.02% at $A = 1.06$ and declines afterwards. The alphas and Sharpe ratios in Panels B and C follow similar patterns. The spread portfolio's alpha in Panel B increases from 0.33 at $A = 0$ to 0.89 at $A = 1.10$. The corresponding Sharpe ratio rises from 0.72 at $A = 0$ to 1.54 at $A = 1.10$, and then declines.

To complete the analysis, Figure 3 displays the performance of the all-anomaly portfolio, where the mean forecast is rewarded for anomaly forecast skewness as follows:

$$AF(x_i)_{j,t} = \mathrm{E}(x_i)_{j,t} + A \times Skew(x_i)_{j,t} \tag{5}$$

In all panels, the long and short portfolios exhibit local maximum and minimum. Consequently, the spread portfolio's mean return peaks at 0.89% ($A = 0.38$), the alpha at 0.51 ($A = 0.44$), and the Sharpe ratio at 0.89 ($A$ = 0.44).

Figures (1)–(3) demonstrate how incorporating the variance, kurtosis, and skewness information into the portfolio construction process improves investing according to the mean forecast. Penalizing stocks for variance and kurtosis risks and rewarding for skewness improves portfolio mean returns, alphas, and Sharpe ratios.

Figure 4 illustrates the performance effect of the trading strategy based on the moments of the anomaly distribution. It illustrates the improvement in cumulative returns when considering the optimal portfolios in Figures (1) – (3). It compares the value of $1 invested according to the monthly forecast mean (serving as the benchmark) with the portfolios from Figures (1) – (3) that yield the highest mean returns: forecast mean minus 0.42 × variance, minus 1.06 × kurtosis, or plus 0.38 × skewness.

Panel A shows the value of the long portfolios compared with the value-weighted market portfolio. An initial $1 investment in the market portfolio at the start of the sample period grew to $187.4 by the end. The mean forecast portfolio accumulates to $760.1—almost four times the market portfolio—demonstrating that investing based on mean anomaly significantly enhances profitability. Incorporating skewness in the calculations yields a portfolio value of $1,079.07. When variance and kurtosis are considered, the portfolio value increases to $1,113.93 and $3,283.73, respectively. These substantial values indicate that incorporating one of the three moments boosts profitability by about 50% with skewness and variance and more than 300% with kurtosis.

Panels B and C present the values of the short and long-minus-short portfolios. The short portfolio values range from $0.32 for the mean–variance portfolio to $2.54 for the mean–kurtosis portfolio. For the long-minus-short portfolios, the mean forecast portfolio value of $43.43 increases to $67.90, $81.67, and $353.99 for the mean–variance, mean–skewness, and mean–kurtosis portfolios. Overall, incorporating these moments significantly enhances profitability in economic terms. Moreover, the stronger impact of kurtosis than the variance implies that arbitrage traders are more worried about negative fat tails than volatility.

## 5. Concluding remarks

We adopt a top-down approach to analyze a particular source of statistical limits to arbitrage: the uncertainty in mispricing implied by anomalies signals. Using 140 well-known anomalies, we estimate the first four moments of the distribution of anomaly forecasts. By quantifying limits to arbitrage based on statistical inferences drawn from these anomaly forecasts, we can measure the risk and reward associated with potential arbitrage opportunities. We interpret the mean and skewness in anomaly distribution forecasts as proxies for potential mispricing and corresponding future abnormal returns. In contrast, the variance and kurtosis in anomaly distribution forecasts represent volatility and downside risk in assessing mispricing, which influences the likelihood of arbitrage trading, and the future realized abnormal returns that arise from this trading. This framework enables us to directly test the economic rationale underlying our proposed statistical limits to arbitrage. Accordingly, we hypothesize that, all else being equal, a lower mean and negative skewness, combined with higher volatility and downside risk, reduce arbitrage trading in the relevant stocks and lead to lower future realized abnormal returns.

The results of our empirical analysis support this hypothesis: higher realized abnormal returns are associated with higher mean forecasts and more positive skewness. In contrast, higher volatility, greater kurtosis, and negative skewness are followed by lower realized abnormal returns. These findings are robust across varying market conditions and subgroups of anomalies with historical profitability. Thus, our proposed source of limits to arbitrage exists independently of pre-investment learning processes, such as identifying anomalies with positive alpha. Notably, the effects of variance and kurtosis have intensified in recent decades. The significant correlations between our measures and future realized returns offer valuable insights into how uncertainty and perceived opportunity in arbitrage profits influence investor behavior and market dynamics.

From a practical perspective, we show that incorporating variance, kurtosis, and skewness into anomalies' portfolio construction improves mean returns, alphas, and Sharpe ratios. The most pronounced improvements stem from mitigating downside risk, suggesting that arbitrageurs are particularly concerned with this type of risk.

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**Table 1. Descriptive statistics**

The table presents the averages of the first four moments of monthly anomaly forecasts. The sample includes all U.S. firms listed on the NYSE, AMEX, and NASDAQ with share codes 10 and 11, including returns of delisting stocks and excluding the NYSE benchmark for the bottom 20% of stocks by market capitalization. Anomalies are percentile-ranked each month to generate anomaly forecasts. Then, we calculate the monthly mean, standard deviation, skewness, and kurtosis of these forecasts for each month-stock. The table displays the averages of these four moments over the entire sample period, spanning from 1971 to 2022. The first row shows the averages of the four moments across all 140 anomalies. The remainder of the table provides the averages of the moments for the 13 clusters of Jensen, Kelly, and Pedersen (2023).

| | Mean | Standard deviation | Skewness | Kurtosis |
|---|---|---|---|---|
| All anomalies | 0.50 | 0.27 | 0.03 | 1.94 |
| | | | | |
| Accruals | 0.50 | 0.16 | 0.02 | 1.79 |
| Debt issuance | 0.50 | 0.23 | 0.00 | 2.07 |
| Investment | 0.50 | 0.21 | 0.02 | 2.23 |
| Low leverage | 0.50 | 0.21 | −0.06 | 2.14 |
| Low risk | 0.50 | 0.21 | 0.07 | 3.21 |
| Momentum | 0.50 | 0.13 | 0.05 | 1.75 |
| Profit growth | 0.50 | 0.24 | −0.01 | 1.82 |
| Profitability | 0.50 | 0.19 | 0.07 | 2.30 |
| Quality | 0.49 | 0.20 | 0.12 | 3.14 |
| Seasonality | 0.50 | 0.26 | −0.01 | 1.93 |
| Size | 0.50 | 0.15 | 0.05 | 1.98 |
| Skewness | 0.50 | 0.17 | 0.00 | 1.73 |
| Value | 0.50 | 0.20 | −0.02 | 2.51 |

**Table 2. Cross-sectional regressions with anomaly forecasts' moments**

The table reports average slopes and their corresponding *t*-statistics (in parentheses) obtained from Fama and MacBeth's (1973) regressions in Equation (1). The sample and the sample period are defined in Table 1. The dependent variable is the stock's future return over 1, 3, 6, or 12 months. The explanatory variables include the mean, variance, skewness, and kurtosis of the stock's 140 anomaly forecasts and control variables. The control variables are the log of the firm's size, book-to-market ratio, idiosyncratic volatility, and stock turnover. Standard errors are based on Bartlett's kernel, which, in turn, implements the Newey–West covariance estimator. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

| Anomaly Forecasts' | $R_{t+1}$ | $R_{t+1:t+3}$ | $R_{t+1:t+6}$ | $R_{t+1:t+12}$ |
|---|---|---|---|---|
| Mean | 0.0551*** | 0.1383*** | 0.2482*** | 0.1523*** |
| | (6.67) | (6.62) | (7.06) | (4.57) |
| Variance | −0.0625*** | −0.1920*** | −0.3301** | −0.2882*** |
| | (−3.22) | (−3.77) | (−3.96) | (−3.37) |
| Skewness | 0.0040*** | 0.0092*** | 0.0165*** | 0.0001 |
| | (3.10) | (2.77) | (2.88) | (0.03) |
| Kurtosis | −0.0042*** | −0.0118*** | −0.0205*** | −0.0162*** |
| | (−5.00) | (−5.41) | (−5.37) | (−4.68) |
| | | | | |
| Size | −0.0005 | −0.0011 | −0.0017 | −0.0011 |
| | (−1.38) | (−1.26) | (−1.12) | (−0.68) |
| Book-to-market | 0.0009 | 0.0022 | 0.0051 | 0.0168*** |
| | (1.07) | (1.08) | (1.33) | (4.03) |
| Idiosyncratic volatility | −0.0332 | −0.0317 | −0.1514 | −0.0313 |
| | (−0.51) | (−0.19) | (−0.55) | (−0.11) |
| Turnover | 0.0001 | 0.0001 | −0.0001 | −0.0012** |
| | (1.07) | (0.27) | (−0.55) | (−2.04) |

**Table 3. Cross-sectional market regressions across time**

The table reports average slopes and their corresponding *t*-statistics (in parentheses) obtained from Fama and MacBeth's (1973) regressions, similar to those in Table 1. The sample and the sample period are defined in Table 1. The dependent variable is the stock's future return over 1, 3, 6, or 12 months. The explanatory variables include the mean, variance, skewness, and kurtosis of the stock's 140 anomaly forecasts and control variables (not tabulated) for the firm's size, value, idiosyncratic risk, and turnover. In Panel A, the sample is divided into two: an early period, that is, 1971–1999; and a recent period, 2000–2022. In Panel B, the sample is limited to anomalies observed only after their discovery. Standard errors are based on Bartlett's kernel, which, in turn, implements the Newey–West covariance estimator. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Panel A. Early and Recent Periods

| | 1971–1999 | | | | 2000–2022 | | | |
|---|---|---|---|---|---|---|---|---|
| | $R_{t+1}$ | $R_{t+1:t+3}$ | $R_{t+1:t+6}$ | $R_{t+1:t+12}$ | $R_{t+1}$ | $R_{t+1:t+3}$ | $R_{t+1:t+6}$ | $R_{t+1:t+12}$ |
| Mean | 0.0775*** | 0.1953*** | 0.3607*** | 0.1814*** | 0.0265** | 0.0653* | 0.1026* | 0.1132** |
| | (8.02) | (8.14) | (9.17) | (4.45) | (1.97) | (1.90) | (1.76) | (2.04) |
| Variance | –0.0401** | –0.1294** | –0.2385** | –0.1795* | –0.0910** | –0.2720*** | –0.4485*** | –0.4344*** |
| | (–1.98) | (–2.55) | (–2.71) | (–1.84) | (–2.54) | (–2.87) | (–2.98) | (–2.90) |
| Skewnes | 0.0080*** | 0.0193*** | 0.0345*** | 0.0074 | –0.0012 | –0.0038 | –0.0069 | –0.0096 |
| | (4.95) | (4.43) | (4.69) | (1.06) | (–0.66) | (–0.78) | (–0.82) | (–1.04) |
| Kurtosis | –0.0025*** | –0.0091*** | –0.0180*** | –0.0122*** | –0.0065*** | –0.0152*** | –0.0239*** | –0.0215*** |
| | (–2.95) | (–4.05) | (–3.86) | (–3.05) | (–4.17) | (–3.96) | (–3.88) | (–3.60) |

Panel B. Post-Discovery Anomalies

| | $R_{t+1}$ | $R_{t+1:t+3}$ | $R_{t+1:t+6}$ | $R_{t+1:t+12}$ |
|---|---|---|---|---|
| Mean | 0.0267*** | 0.0102 | 0.0064 | 0.0428 |
| | (3.40) | (1.30) | (0.82) | (1.35) |
| Variance | –0.0230** | –0.0303*** | –0.0262*** | –0.1068** |
| | (–2.29) | (–3.06) | (–2.77) | (–2.38) |
| Skewness | 0.0012 | –0.0001 | –0.0015 | –0.0067 |
| | (1.15) | (–0.13) | (–1.44) | (–1.43) |
| Kurtosis | –0.0029*** | –0.0031*** | –0.0029*** | –0.0137*** |
| | (–3.98) | (–4.05) | (–4.08) | (–4.64) |

**Table 4. Market conditions**

The table reports average slopes and their corresponding *t*-statistics (in parentheses) obtained from Fama and MacBeth's (1973) regressions, similar to those in Table 2. The sample and the sample period are defined in Table 1. The dependent variable is the stock's future return over 1, 3, 6, or 12 months. The explanatory variables include the mean, variance, skewness, and kurtosis of the stock's 140 anomaly forecasts and control variables (not tabulated) for the firm's size, value, idiosyncratic risk, and turnover. The analysis is implemented as follows. Panel (A): positive or negative sentiment per Baker and Wurgler (2006); Panel (B): market volatility is above or below the median market volatility in previous months; Panel (C): illiquidity is above or below the median market illiquidity in previous months per Amihud (2002); and Panel (D): market 2-year return is positive or negative per Cooper, Gutierrez, and Hameed (2004). Standard errors are based on Bartlett's kernel, which, in turn, implements the Newey–West covariance estimator. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Panel A. High versus Low Sentiment

| | Low Sentiment | | | | High Sentiment | | | |
|---|---|---|---|---|---|---|---|---|
| | $R_{t+1}$ | $R_{t+1:t+3}$ | $R_{t+1:t+6}$ | $R_{t+1:t+12}$ | $R_{t+1}$ | $R_{t+1:t+3}$ | $R_{t+1:t+6}$ | $R_{t+1:t+12}$ |
| Mean | 0.0475*** | 0.1013*** | 0.1985*** | 0.1412*** | 0.1523*** | 0.1673*** | 0.2862**** | 0.1621*** |
| | (4.23) | (3.82) | (4.33) | (3.56) | (4.57) | (5.60) | (5.73) | (3.20) |
| Variance | −0.0227 | −0.0717 | −0.1622 | −0.2888*** | −0.2882*** | −0.3137*** | −0.5057*** | −0.2879** |
| | (−0.97) | (−1.21) | (−1.56) | (−2.61) | (−3.37) | (−4.13) | (−4.30) | (−2.39) |
| Skewnes | 0.0054*** | 0.0124*** | 0.0277*** | 0.0154** | 0.0001 | 0.0052 | 0.0052 | −0.0135 |
| | (2.99) | (2.67) | (3.36) | (2.18) | (0.03) | (1.17) | (0.72) | (−1.62) |
| Kurtosis | −0.0018* | −0.0064** | −0.0138*** | −0.0177*** | −0.0162*** | −0.0163*** | −0.0265*** | −0.0141*** |
| | (−1.82) | (−2.45) | (−2.64) | (−3.91) | (−4.68) | (−5.35) | (−5.52) | (−2.87) |

Panel B. High versus Low Volatility

| | Low Volatility | | | | High Volatility | | | |
|---|---|---|---|---|---|---|---|---|
| | $R_{t+1}$ | $R_{t+1:t+3}$ | $R_{t+1:t+6}$ | $R_{t+1:t+12}$ | $R_{t+1}$ | $R_{t+1:t+3}$ | $R_{t+1:t+6}$ | $R_{t+1:t+12}$ |
| Mean | 0.0594*** | 0.1525*** | 0.2825*** | 0.1828*** | 0.0516*** | 0.1258*** | 0.2180**** | 0.1248*** |
| | (6.07) | (6.88) | (7.63) | (4.17) | (3.95) | (4.09) | (4.15) | (2.85) |
| Variance | −0.0021 | −0.1179** | −0.2266*** | −0.3026*** | −0.1185*** | −0.2566*** | −0.4212*** | −0.2752** |
| | (−0.10) | (−2.56) | (−2.78) | (−3.16) | (−3.75) | (−3.32) | (−3.34) | (−2.12) |
| Skewnes | 0.0052*** | 0.0104*** | 0.0211*** | 0.0077 | 0.0028 | 0.0081 | 0.0124 | −0.0066 |
| | (3.19) | (2.88) | (3.22) | (1.08) | (1.53) | (1.79) | (1.54) | (−0.84) |
| Kurtosis | −0.0015 | −0.0093*** | −0.0148*** | −0.0136*** | −0.0067*** | −0.0140*** | −0.0256*** | −0.0185*** |
| | (−1.60) | (−4.64) | (−4.42) | (−3.46) | (−4.86) | (−4.16) | (−4.17) | (−3.64) |

Panel C. High versus Low Illiquidity

| | Low Illiquidity | | | | High Illiquidity | | | |
|---|---|---|---|---|---|---|---|---|
| | $R_{t+1}$ | $R_{t+1:t+3}$ | $R_{t+1:t+6}$ | $R_{t+1:t+12}$ | $R_{t+1}$ | $R_{t+1:t+3}$ | $R_{t+1:t+6}$ | $R_{t+1:t+12}$ |
| Mean | 0.0555*** | 0.1417*** | 0.2513*** | 0.1653*** | 0.0517* | 0.1018 | 0.2151* | 0.0164 |
| | (6.41) | (6.55) | (76.96 | (4.67) | (1.80) | (1.41) | (1.73) | (0.19) |
| Variance | −0.0613*** | −0.1915** | −0.3360*** | −0.2655*** | −0.0977* | −0.1971 | −0.2670 | −0.5255** |
| | (−2.98) | (−3.55) | (−3.81) | (−2.92) | (−1.87) | (−1.53) | (−1.17) | (−2.31) |
| Skewness | 0.0036*** | 0.0092*** | 0.0159*** | −0.0001 | 0.0075** | 0.0093 | 0.0227 | 0.0028 |
| | (2.69) | (2.61) | (2.63) | (−0.02) | (1.97) | (1.00) | (1.44) | (0.24 |
| Kurtosis | −0.0047*** | −0.0130*** | −0.0230*** | −0.0173*** | −0.0005 | −0.0005 | 0.0053 | −0.0045 |
| | (−5.20) | (−5.63) | (−5.66) | (−4.66) | (−0.21) | (0.10) | (0.71) | (−0.72) |

Panel D. High versus Low Market

| | Low Market | | | | High Market | | | |
|---|---|---|---|---|---|---|---|---|
| | $R_{t+1}$ | $R_{t+1:t+3}$ | $R_{t+1:t+6}$ | $R_{t+1:t+12}$ | $R_{t+1}$ | $R_{t+1:t+3}$ | $R_{t+1:t+6}$ | $R_{t+1:t+12}$ |
| Mean | 0.0530** | 0.0982 | 0.2112** | 0.1533** | 0.0555*** | 0.1443*** | 0.2538*** | 0.1521*** |
| | (2.14) | (1.60) | (2.10) | (1.98) | (6.21) | (6.49) | (6.81) | (4.19) |
| Variance | −0.0669 | −0.2101 | −0.3124 | −0.3272 | −0.0640*** | −0.1892*** | −0.3327*** | −0.2822*** |
| | (−1.05) | (−1.23) | (−1.02) | (−1.06) | (−3.22) | (−3.68) | (−4.02) | (−3.34) |
| Skewness | 0.0009 | −0.0023 | 0.0138 | 0.0258*** | 0.0044*** | 0.0109*** | 0.0169*** | −0.0038 |
| | (0.30) | (−0.38) | (1.61) | (2.90) | (3.08) | (2.94) | (2.63) | (−0.61) |
| Kurtosis | −0.0048* | −0.0113 | −0.0131 | −0.0095 | −0.0042*** | −0.0119*** | −0.0217 | −0.0172*** |
| | (−1.65) | (−1.53) | (−1.20) | (−0.87) | (−4.91) | (−5.39) | (−5.40) | (−4.85) |

**Table 5. Clusters' contribution to statistical limits to arbitrage**

The table reports average slopes, and their corresponding *t*-statistics (in parentheses) obtained from Fama and MacBeth's (1973) regressions in Equation (2). The sample and the sample period are defined in Table 1. The dependent variable is the stock's future return over one month. The explanatory variables in each regression include the mean, variance, skewness, and kurtosis of all 140 anomalies, along with the moments of anomalies within each cluster. Each row represents the results for one of the 13 clusters. The control variables (not tabulated) are for the firm's size, value, idiosyncratic risk, and turnover. Standard errors are based on Bartlett's kernel, which, in turn, implements the Newey–West covariance estimator. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

| | All Anomalies | | | | Within Cluster | | | |
|---|---|---|---|---|---|---|---|---|
| **Cluster** | **Mean** | **Variance** | **Skewness** | **Kurtosis** | **Mean** | **Variance** | **Skewness** | **Kurtosis** |
| Accruals | 0.0651*** | −0.0279 | 0.0045*** | −0.0039*** | 0.0004 | 0.0055 | −0.0003 | −0.0001 |
| | (5.85) | (−0.81) | (2.81) | (−3.09) | (0.27) | (0.94) | (−1.12) | (−0.31) |
| Debt | 0.0657*** | −0.0254 | 0.0045*** | −0.0034*** | −0.0011 | 0.0069 | −0.0001 | −0.0003* |
| | (5.41) | (−0.88) | (2.71) | (−2.70) | (−0.40) | (1.21) | (−0.41) | (−1.88) |
| Investment | 0.0596*** | −0.0713** | 0.0048*** | −0.0043*** | −0.0007 | 0.0249*** | −0.0006* | 0.0002 |
| | (5.92) | (−2.41) | (3.64) | (−4.41) | (−0.33) | (3.80) | (−1.79) | (1.37) |
| Low | 0.0601*** | −0.0613*** | 0.0050*** | −0.0042*** | −0.0026 | 0.0260*** | −0.0002 | 0.0007*** |
| | (6.09) | (−2.63) | (3.99) | (−4.87) | (−0.84) | (4.20) | (−0.61) | (4.02) |
| Low risk | 0.0731*** | −0.0460* | 0.0036*** | −0.0046*** | −0.0084 | −0.0475*** | 0.0005** | −0.0002** |
| | (9.78) | (−1.86) | (3.08) | (−4.84) | (−1.64) | (−5.25) | (2.13) | (−2.06) |
| Momentum | 0.0589*** | −0.0658** | 0.0040*** | −0.0045*** | 0.0032 | 0.0015 | −0.0013*** | 0.0008** |
| | (6.49) | (−2.27) | (3.21) | (−4.74) | (1.16) | (0.17) | (−3.73) | (2.18) |
| Profit growth | 0.0595*** | −0.0604** | 0.0043*** | −0.0041*** | 0.0031 | 0.0178*** | 0.0001 | 0.0006*** |
| | (6.28) | (−2.08) | (3.28) | (−4.33) | (1.46) | (2.78) | (0.20) | (3.11) |
| Profitability | 0.0629*** | −0.0651** | 0.0047*** | −0.0041*** | −0.0017 | 0.0175*** | −0.0001 | 0.0003** |
| | (6.39) | (−2.08) | (3.44) | (−4.08) | (−1.11) | (3.07) | (−0.28) | (2.05) |
| Quality | 0.0609*** | −0.0253 | 0.0046*** | −0.0033*** | 0.0048 | 0.0066 | 0.0005 | −0.0003*** |
| | (4.78) | (−0.72) | (2.83) | (−2.62) | (1.55) | (0.93) | (1.45) | (−2.58) |
| Seasonality | 0.065***9 | −0.0719** | 0.0051*** | −0.0046*** | −0.0082*** | 0.0374*** | −0.0004 | 0.0009*** |
| | (6.69) | (−2.35) | (3.95) | (−4.75) | (−3.63) | (5.78) | (−1.18) | (4.14) |
| Size | 0.0583*** | −0.0617*** | 0.0044*** | −0.0046*** | 0.0024 | −0.0009 | 0.0006** | 0.0009** |
| | (5.77) | (−2.04) | (3.38) | (−4. 69) | (1.16) | (−0.11) | (2.00) | (2.53) |
| Skewness | 0.0546*** | −0.0559* | 0.0041*** | −0.0045*** | 0.0060*** | −0.0102** | 0.0010**** | −0.0010*** |
| | (5.50) | (−1.87) | (3.07) | (−4.53) | (3.61) | (−2.13) | (4.13) | (−3.72) |
| Value | 0.0582*** | −0.0601** | 0.0044*** | −0.0042*** | −0.0006 | 0.0237** | −0.0002 | 0.0005*** |
| | (6.08) | (−2.20) | (3.54) | (−4.67) | (0.18) | (2.29) | (−0.59) | (4.00) |

**Table 6. Cross-sectional regressions: Positive-alpha anomalies**

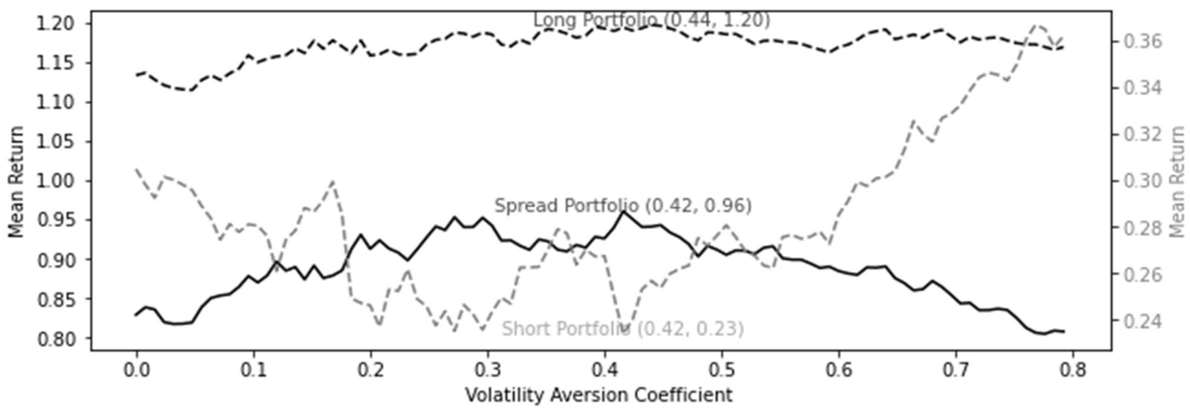
The table reports average slopes and *t*-statistics (in parentheses) obtained from Fama and MacBeth's (1973) regressions in Equation (1). The sample and the sample period are defined in Table 1. The dependent variable is the stock's future return over 1, 3, 6, or 12 months. The explanatory variables include the mean, variance, skewness, and kurtosis of the stock's positive-alpha anomaly forecasts and control variables. Positive-alpha anomalies include anomalies that previously yielded abnormal profits. An anomaly is included in the four-moment calculations at time *t* only if regressing the anomaly's zero-cost portfolio on the five Fama–French (2015) factors yield a positive alpha at time $t - 1$. The control variables are the log of the firm's size, book-to-market ratio, idiosyncratic volatility, and stock turnover. Standard errors are based on Bartlett's kernel, which, in turn, implements the Newey–West covariance estimator. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

| Anomaly Forecasts' | $R_{t+1}$ | $R_{t+1:t+3}$ | $R_{t+1:t+6}$ | $R_{t+1:t+12}$ |
|---|---|---|---|---|
| Mean | 0.0452*** | 0.1203*** | 0.2095*** | 0.1030** |
| | (6.24) | (6.44) | (6.62) | (2.43) |
| Variance | −0.0428*** | −0.1331*** | −0.2342*** | −0.2353*** |
| | (−2.62) | (−3.13) | (−3.36) | (−3.08) |
| Skewness | 0.0030*** | 0.0075*** | 0.0134*** | −0.0019 |
| | (2.91) | (2.89) | (3.10) | (−0.41) |
| Kurtosis | −0.0028*** | −0.0077*** | −0.0124*** | −0.0084*** |
| | (−5.92) | (−6.05) | (−5.60) | (−3.78) |
| | | | | |
| Size | −0.0009*** | −0.0023*** | −0.0039** | −0.0027* |
| | (−2.74) | (−2.60) | (−2.51) | (−1.68) |
| Book-to-market | 0.0019** | 0.0050** | 0.0099*** | 0.0210*** |
| | (2.39) | (2.32) | (2.66) | (5.08) |
| Idiosyncratic volatility | −0.0721 | −0.1250 | −0.3187 | −0.2327 |
| | (−1.02) | (−0.68) | (−1.07) | (−0.74) |
| Turnover | 0.0000 | −0.0001 | −0.0005 | −0.0015*** |
| | (0.37) | (−0.46) | (−0.98) | (−2.69) |

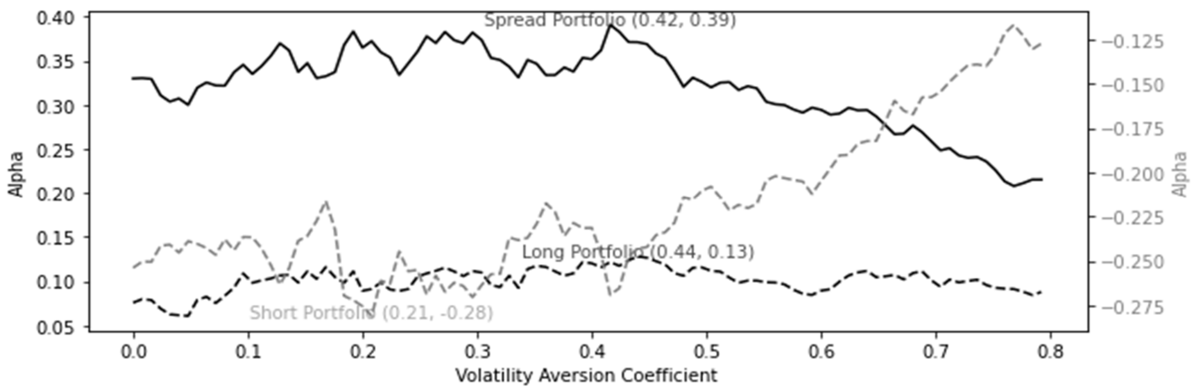
**Figure 1. Impact of variance of anomaly forecasts on realized returns**

The figure illustrates the performance of long, short, and zero-cost spread (long-minus-short) portfolios, where stocks are sorted based on mean anomaly forecasts and discounted for the variance of these forecasts. The sample and the sample period are defined in Table 1. The long and short portfolios consist of stocks from the top and bottom deciles sorted by the all-anomaly forecast. The all-anomaly forecast $AF_{j,t}\ (x_i)$ is calculated as $E(x_i)_{j,t} - A \times \sigma^2(x_i)_{j,t}$, where $E(x_i)_{j,t}$ is the percentile-rank mean forecast across all 140 anomalies per stock, $A$ is a variance-aversion coefficient, and $\sigma^2(x_i)_{j,t}$ represents the monthly percentile-ranked variance of anomaly forecasts per stock. Panels A, B, and C display the monthly mean returns, monthly alphas from regressing these returns on the Fama–French five factors, and the corresponding Sharpe ratios for the three portfolios. The numbers in parentheses indicate the coordinates of local maxima or minima: the first number is the maximal (minimal) value of $A$ on the $X$-axis, and the second number is the corresponding $Y$-axis value of the portfolio. The horizontal axis on the right-hand side corresponds to the short portfolios.

Panel A. Monthly Mean Returns




Panel B. Portfolio Alpha




Panel C. Sharpe Ratio

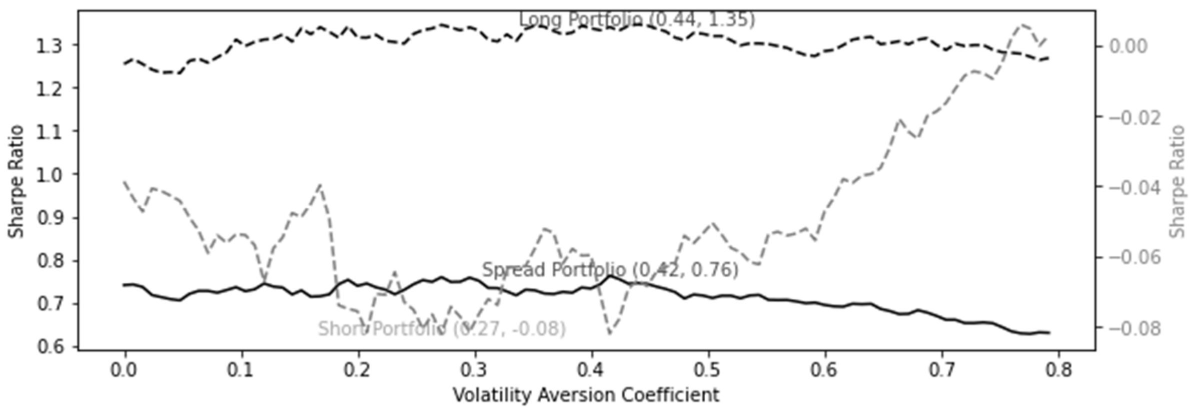
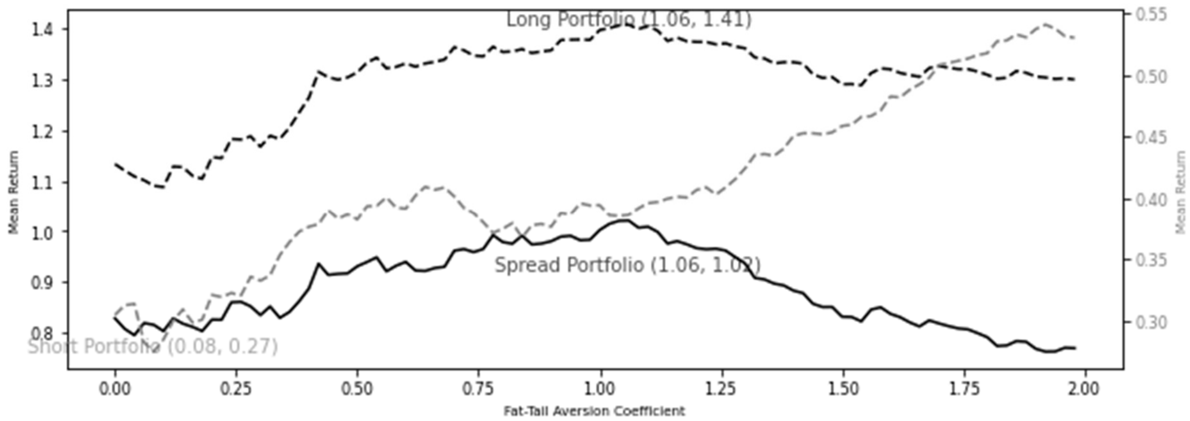


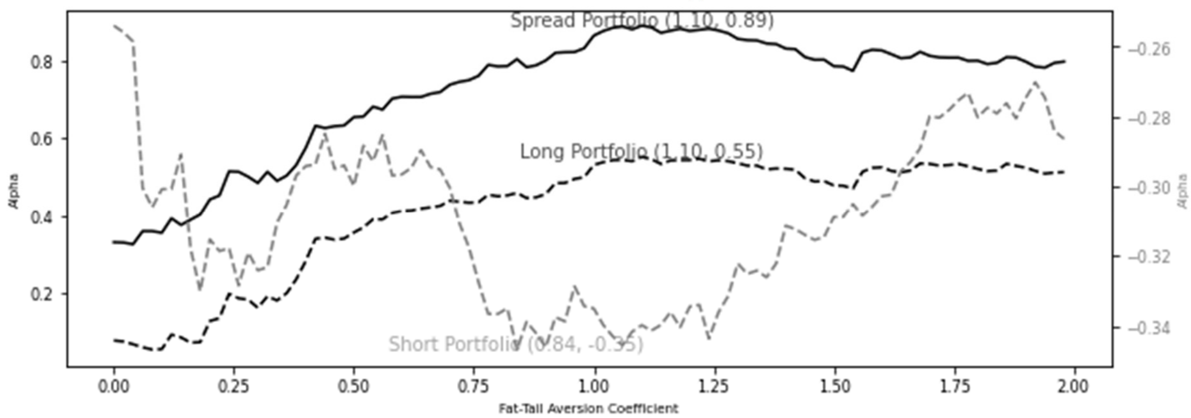
**Figure 2. Impact of kurtosis of anomaly forecasts on realized returns**

The figure illustrates the performance of long, short, and zero-cost spread (long-minus-short) portfolios, where stocks are sorted based on mean anomaly forecasts discounted for the kurtosis of these forecasts. The sample and the sample period are defined in Table 1. The long and short portfolios consist of stocks from the top and bottom deciles sorted by the all-anomaly forecast. The all-anomaly forecast $AF_{j,t}$ ($x_i$) is calculated as $E(x_i)_{j,t}$ $-A \times Kurt(x_i)_{j,t}$, where $E(x_i)_{j,t}$ is the monthly percentile-ranked forecast across all 140 anomalies per stock, $A$ is a kurtosis-aversion coefficient, and $Kurt(x_i)_{j,t}$ represents the monthly percentile-ranked kurtosis of anomaly forecasts per stock. Panels A, B, and C display the monthly mean returns, monthly alphas from regressing these returns on the Fama–French five factors, and the corresponding Sharpe ratios for the three portfolios. The numbers in parentheses indicate the coordinates of local maxima or minima: The first number is the maximal (minimal) value of $A$ on the $X$-axis, and the second number is the corresponding $Y$-axis value of the portfolio. The horizontal axis on the right-hand side corresponds to the short portfolios.

Panel A. Monthly Mean Returns




Panel B. Portfolio Alpha




Panel C. Sharpe Ratio

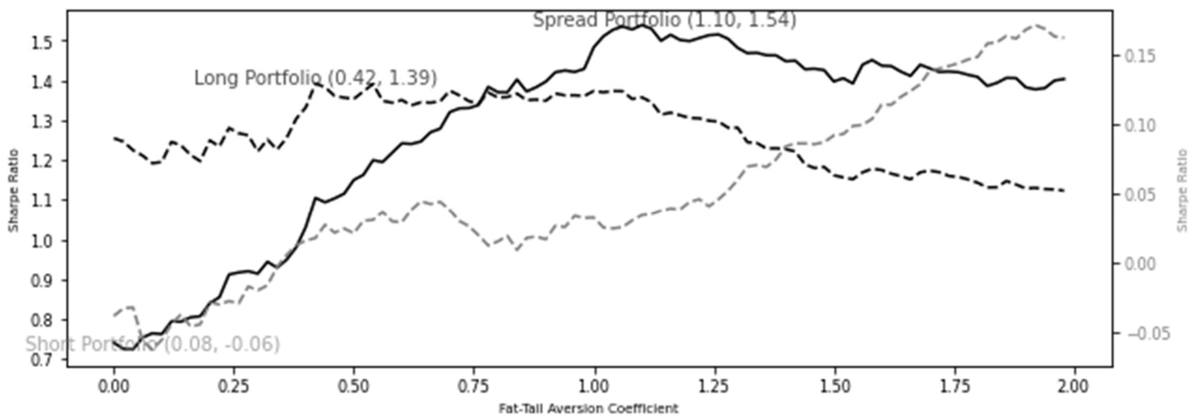
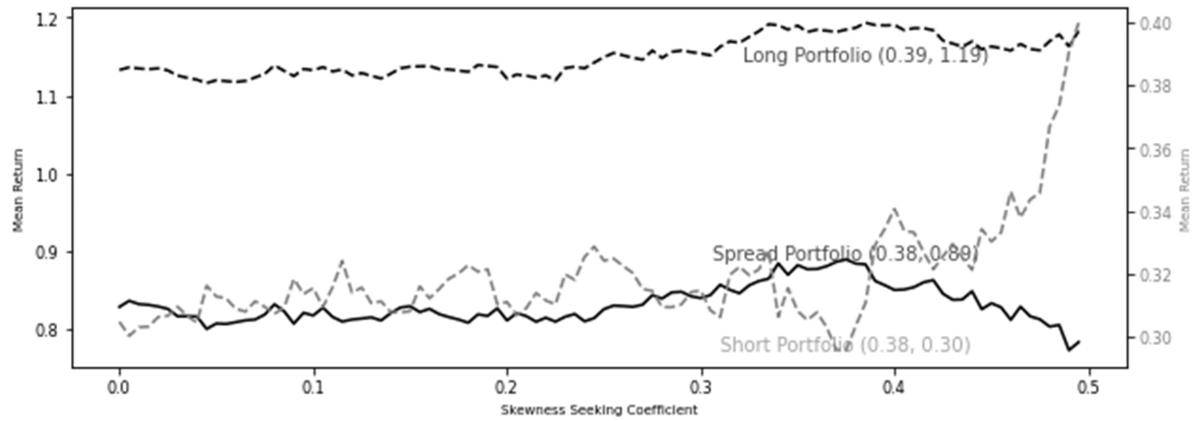


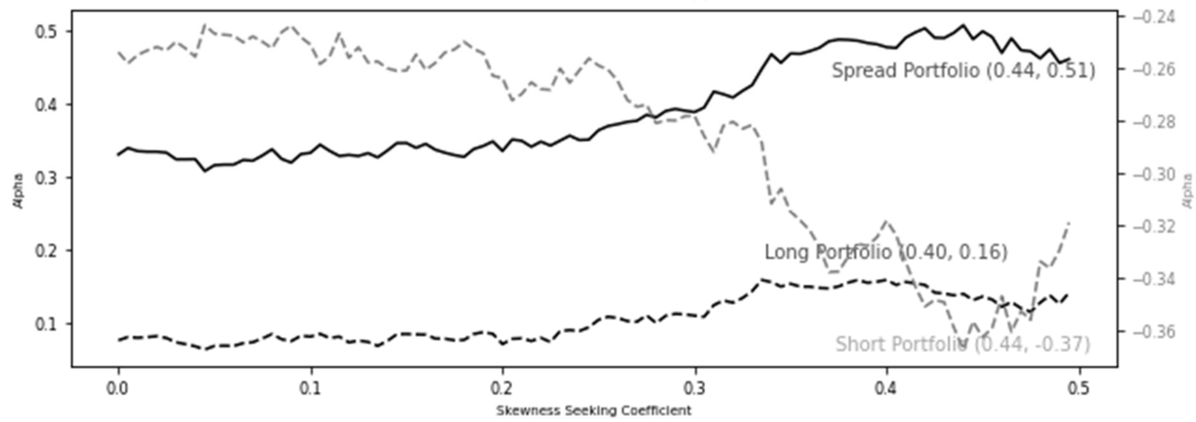
**Figure 3. Impact of skewness of anomaly forecasts on realized returns**

The figure illustrates the performance of long, short, and zero-cost spread (long-minus-short) portfolios, where stocks are sorted based on mean anomaly forecasts and elevated for the skewness of these forecasts. The sample and the sample period are defined in Table 1. The long and short portfolios consist of stocks from the top and bottom deciles sorted by the all-anomaly forecast. The all-anomaly forecast $AF_{j,t}(x_i)$ is calculated as $E(x_i)_{j,t}$ + $A \times Skew(x_i)_{j,t}$, where $E(x_i)_{j,t}$ is the monthly percentile-ranked forecast across all 140 anomalies per stock, $A$ is a skew-preference coefficient, and $Skew(x_i)_{j,t}$ represents the monthly percentile-ranked skewness of anomaly forecasts per stock. Panels A, B, and C display the monthly mean returns, monthly alphas from regressing these returns on the Fama–French five factors, and the corresponding Sharpe ratios for the three portfolios. The numbers in parentheses indicate the coordinates of local maxima or minima: the first number is the maximal (minimal) value of $A$ on the $X$-axis, and the second number is the corresponding $Y$-axis value of the portfolio. The horizontal axis on the right-hand side corresponds to the short portfolios.

Panel A. Monthly Mean Returns




Panel B. Portfolio Alpha




Panel C. Sharpe Ratio

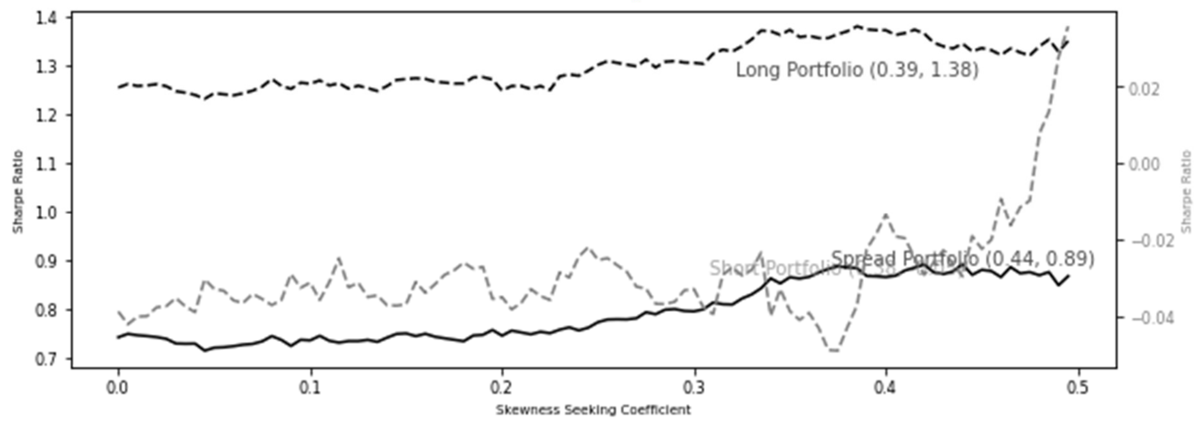
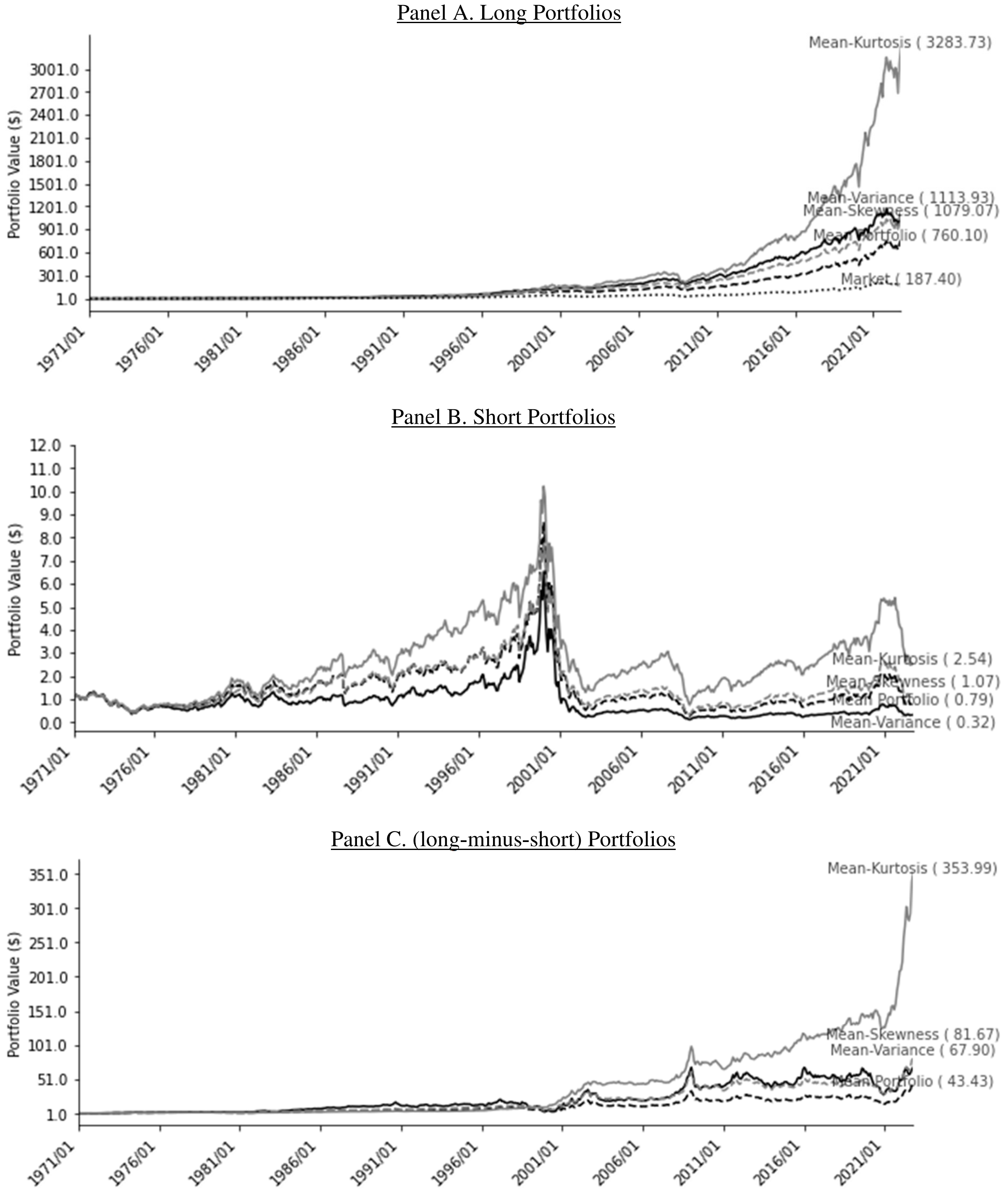


**Figure 4. Comparing performance with moments**

The figure illustrates how forecasts' moments improve investment performance. The sample and the sample period are defined in Table 1. The figure compares the cumulative values from investing $1 at the beginning of the sample period according to the monthly forecast mean, forecast mean minus 0.42 × variance, forecast mean minus 1.06 × kurtosis, and mean plus 0.38 × skewness. Panel A shows the cumulative value of the long portfolios versus the market portfolio. Panel B and C correspond to the short and the long-minus-short portfolios.



## Appendix A. List of characteristics

The table describes the 140 anomalies used in constructing the investment portfolio. Jensen, Kelly, and Pedersen (2023) offer a comprehensive framework for anomaly computation. They also classify these anomalies into 13 clusters.

| Characteristic name | Description | Cluster |
|---|---|---|
| age | Age | Low leverage |
| aliq_mat | Liquidity scaled by lagged Market Assets | Low leverage |
| aliq_at | Liquidity scaled by lagged Assets | Investment |
| ami_126d | Amihud (2002) Measure | Size |
| at_be | Book Leverage | Low leverage |
| at_gr1 | Asset Growth | Investment |
| at_me | Asset to Market Equity | Value |
| at_turnover | Asset Turnover | Quality |
| be_gr1a | Book Equity Change 1 yr scaled by Assets | Investment |
| be_me | Book Equity scaled by Market Equity | Value |
| beta_60m | 60 Month CAPM Beta | Low risk |
| beta_capm_21d | CAPM Beta 21 days | Low risk |
| beta_capm_252d | CAPM Beta 252 days | Low risk |
| betabab_1260d | Betting Against Beta | Low risk |
| betadown_252d | Downside Beta | Low risk |
| bev_mev | Book Enterprise Value scaled by Market Equity | Value |
| bidaskhl_21d | 21 Day Bid–Ask High–Low | Low leverage |
| capex_abn | Abnormal Corporate Investment | Debt issuance |
| capx_gr1 | CAPX 1-year growth | Investment |
| capx_gr2 | CAPX 2-year growth | Investment |
| capx_gr3 | CAPX 3-year growth | Investment |
| cash_at | Cash and Short-Term Investments scaled by Assets | Low leverage |
| chcsho_12m | Change in Shares – 12 Month | Value |
| coa_gr1a | Inventory Change | Investment |
| col_gr1a | Current Operating Liabilities Change | Investment |
| cop_at | Cash Based Operating Profitability scaled by Assets | Quality |
| cop_atl1 | Cash Based Operating Profitability scaled by lagged Assets | Quality |
| corr_1260d | Correlation to Market | Seasonality |
| coskew | Coskewness | Seasonality |
| cowc_gr1a | Change in current operating working capital | Accruals |
| dbnetis_at | Net Long-Term Debt Issuance scaled by Assets | Seasonality |
| debt_gr3 | Total Debt Growth 3yr | Debt issuance |
| debt_me | Total Debt scaled by Market Equity | Value |
| dgp_dsale | Change Gross Profit minus Change Sales | Quality |
| div12m_me | Dividend to Price – 12 Months | Value |
| dolvol_126d | Dollar Volume | Size |
| dolvol_var_126d | Dollar Volume Volatility | Profitability |
| dsale_dinv | Change Sales minus Change Inventory | Profit growth |
| dsale_drec | Change Sales minus Change Receivables | Profit growth |
| dsale_dsga | Change Sales minus Change SG&A | Profit growth |
| earnings_variability | Earnings Variability | low risk |
| ebit_bev | Operating Profit after Depreciation scaled by Book Enterprise V alue | Profitability |
| ebit_sale | Operating Profit Margin after Depreciation | Profitability |
| ebitda_mev | Operating Profit before Depreciation scaled by MEV | Value |
| emp_gr1 | Employee Growth | Investment |
| eqnetis_at | Equity Net Issuance scaled by Assets | Value |
| eqnpo_12m | Net Equity Payout – 12 Month | Value |
| eqnpo_me | Equity Net Payout scaled by Market Equity | Value |
| eqpo_me | Net Equity Payout scaled by Market Equity | Value |
| f_score | Piotroski *F*-Score | Profitability |
| fcf_me | Free Cash Flow scaled by Market Equity | Value |
| fnl_gr1a | Financial Liabilities Change | Debt issuance |

| | | |
|---|---|---|
| gp_at | Gross Profit scaled by Assets | Quality |
| gp_atl1 | Gross Profit scaled by lagged Assets | Quality |
| inv_gr1 | Inventory Change | Investment |
| inv_gr1a | Inventory Change 1 yr | Investment |
| inv_gr1Q | Inventory Change 1 qr | Investment |
| iskew_capm_21d | CAPM Skewness 21 days | Short-term reversal |
| ivol_capm_21d | CAPM Idiosyncratic Vol. 21 days | Low risk |
| ivol_capm_252d | CAPM Idiosyncratic Vol. 252 days | Low risk |
| ivol_capm_60m | CAPM Idiosyncratic Vol. 60 months | Low risk |
| ivol_ff3_21d | Fama and French Idiosyncratic Vol. | Low risk |
| kz_index | Kaplan–Zingales Index | Seasonality |
| lti_gr1a | Change in long-term investments | Seasonality |
| lnoa_gr1a | Change in Long-Term NOA scaled by average Assets | Investment |
| mispricing_mgmt | Management-Based Mispricing | investment |
| mispricing_perf | Performance-Based Mispricing | Quality |
| ncoa_gr1a | Noncurrent Operating Assets Change | Investment |
| ncol_gr1a | Noncurrent Operating Liabilities | Debt issuance |
| netdebt_me | Net Debt scaled by Market Equity | Low leverage |
| netis_at | Net total issuance | Value |
| nfna_gr1a | Net Financial Assets Change | Debt issuance |
| ni_ar1 | 1 yr lagged Net Income to Assets | Debt issuance |
| niq_at | Quarterly Income scaled by Assets | Quality |
| ni_be | Net Income scaled by Book Equity | Profitability |
| ni_inc8q | Number of Consecutive Earnings Increases | Quality |
| ni_me | Net Income scaled by Market Equity | Value |
| niq_be | Quarterly Return on Equity | Profitability |
| niq_su | Earnings Surprise | Profit growth |
| nncoa_gr1a | Net Noncurrent Operating Assets Change | Investment |
| noa_at | Net Operating Assets to Assets | Debt issuance |
| noa_gr1a | Net Operating Assets Change | Investment |
| o_score | Ohlson O-Score | Profitability |
| oaccruals_at | Operating Accruals | Accruals |
| oaccruals_ni | Percent Operating Accruals | Accruals |
| ocf_at | Operating Cash Flow scaled by Assets | Profitability |
| ocf_at_chg1 | Change in Operating Cash Flow scaled by Assets | Profit growth |
| ocf_me | Operating Cash Flow to Assets scaled by Market Equity | Value |
| op_at | Ball Operating Profit to Assets | Quality |
| op_atl1 | Ball Operating Profit scaled by lagged Assets | Quality |
| ope_be | Operating Profit to Equity scaled by Book Equity | Profitability |
| ope_bel1 | Operating Profit scaled by lagged Book Equity | Profitability |
| opex_at | Operating Leverage | Quality |
| pi_nix | Earnings before Tax and Extraordinary Items to Net Income Incl uding Extraordinary Items | Seasonality |
| ppeinv_gr1a | Change in Property, Plant and Equipment Less Inventories scale d by lagged Assets | Investment |
| prc | Stock price | Size |
| prc_highprc_252d | Price-to-High 252 days | Momentum |
| qmj_prof | Quality Minus Junk – Profit | Quality |
| qmj_safety | Quality Minus Junk – Safety | Quality |
| R1 | Short-Term Reversal | short-term reversal |
| R1360 | Momentum 13–60 Month | Investment |
| R16 | Momentum 1–6 Months | Momentum |
| R212 | Momentum 2–12 Months | Momentum |
| R712 | Momentum 7–12 Months | Profit growth |
| rd_me | R&D scaled by Market Equity | Size |
| rd_sale | R&D scaled by Sales | Low leverage |
| rd5_at | R&D Capital-to-Assets | Low leverage |
| rmax1_21d | Maximum Return | Low risk |
| rmax5_21d | Mean Maximum Return 21 days | Low risk |

| | | |
|---|---|---|
| rmax5_rvol_21d | Max Return to Volatility | Short-term reversal |
| rskew_21d | Return Skewness | Short-term reversal |
| rvol_21d | Return Volatility 21 days | Low risk |
| rvol_252d | Return Volatility 252 days | Low risk |
| sale_bev | Sales scaled by Book Enterprise Value | Quality |
| sale_emp_gr1 | Sales scaled by Employees Growth | Profit growth |
| sale_gr1 | Sales Growth 1yr | Investment |
| sale_gr3 | Sales Growth 3yr | Investment |
| sale_me | Sales Growth scaled by Market Equity | Value |
| saleq_su | Revenue Surprise | Profit growth |
| seas_1_1na | Year 1-lagged return, non-annual | Momentum |
| seas_1_1an | Year 1-lagged return, annual | Profit growth |
| seas_11_15an | 11–15 Year Annual Seasonality | Seasonality |
| seas_11_15na | 11–15 Year Non-Annual Seasonality | Seasonality |
| seas_2_5an | 2–5 Year Annual Seasonality | Seasonality |
| seas_2_5na | 2–5 Year Non-Annual Seasonality | Investment |
| seas_6_10an | 6–10 Year Annual Seasonality | Seasonality |
| seas_6_10na | 6–10 Year Non-Annual Seasonality | low risk |
| size | Market Equity | Size |
| sti_gr1a | Change in Short-Term Investments scaled by Assets | Seasonality |
| taccruals_at | Total Accruals | Accruals |
| taccruals_ni | Percent Total Accruals | Accruals |
| tangibility | Tangibility | low leverage |
| tax_gr1a | Effective Tax Rate Change | Profit growth |
| turnover_126d | Turnover 126 days | Low risk |
| turnover_21d | Turnover 21 days | Low risk |
| turnover_var_126d | Turnover Volatility | Profitability |
| z_score | Altman Z-Score | Low leverage |
| zero_trades_126d | Zero Trades 126 days | Low risk |
| zero_trades_21d | Zero Trades 21 days | Low risk |
| zero_trades_252d | Zero Trades 252 days | Low risk |

## Appendix B. Clusters' contribution to statistical limits to arbitrage

The table reports average slopes, and their corresponding *t*-statistics (in parentheses) obtained from Fama and MacBeth's (1973) regressions in Equation (2). The dependent variable is the stock's future return over 3, 6, or 12 months. The explanatory variables in each regression include the mean, variance, skewness, and kurtosis of all 140 anomalies, along with the moments of anomalies of one of the 13 clusters. The control variables (not tabulated) are for the firm's size, value, idiosyncratic risk, and turnover. Standard errors are based on Bartlett's kernel, which, in turn, implements the Newey–West covariance estimator. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Panel A. Realized Returns over Months 1–3

| | All Anomalies | | | | Within Cluster | | | |
|---|---|---|---|---|---|---|---|---|
| **Cluster** | **Mean** | **Variance** | **Skewness** | **Kurtosis** | **Mean** | **Variance** | **Skewness** | **Kurtosis** |
| Accruals | 0.1647*** | −0.0990 | 0.0111*** | −0.0111*** | 0.0021 | 0.0180 | −0.0009 | 0.0001 |
| | (5.71) | (−1.06) | (2.66) | (−3.34) | (0.59) | (1.15) | (−1.33) | (0.10) |
| Debt issuance | 0.1665*** | −0.0940 | 0.0112** | −0.0099*** | −0.0028 | 0.0166 | −0.0009 | −0.0009* |
| | (5.26) | (−1.21) | (2.56) | (−3.00) | (−0.35) | (1.10) | (−1.36) | (−1.90) |
| Investment | 0.1486*** | −0.2191*** | 0.0110*** | −0.0119*** | −0.0005 | 0.0763*** | −0.0019 | 0.0004 |
| | (5.73) | (−2.73) | (3.18) | (−4.59) | (−0.09) | (4.74) | (−2.09) | (1.20) |
| Low | 0.1504*** | −0.1868*** | 0.0116*** | −0.0117*** | −0.0080 | 0.0740*** | −0.0005 | 0.0022*** |
| | (5.98) | (−2.94) | (3.67) | (−5.19) | (−0.96) | (4.57) | (−0.47) | (4.41) |
| Low risk | 0.1896*** | −0.1517** | 0.0087*** | −0.0128*** | −0.0261 | −0.1261*** | 0.0008 | −0.0006** |
| | (10.03) | (−2.33) | (2.92) | (−5.14) | (−1.89) | (−5.54) | (1.18) | (−2.01) |
| Momentum | 0.1440*** | −0.1916** | 0.0097*** | −0.0126*** | 0.0195 | 0.0015 | −0.0034*** | 0.0023** |
| | (6.09) | (−2.45) | (2.95) | (−5.00) | (2.81) | (0.07) | (−4.43) | (2.45) |
| Profit growth | 0.1496*** | −0.1801** | 0.0100*** | −0.0118*** | 0.0053 | 0.0354** | −0.0001 | 0.0019*** |
| | (6.09) | (−2.28) | (2.90) | (−4.71) | (1.08) | (2.28) | (−0.08) | (3.52) |
| Profitability | 0.1591*** | −0.2019** | 0.0111*** | −0.0116*** | −0.0036 | 0.0528*** | 0.0000 | 0.0008** |
| | (6.24) | (−2.37) | (3.03) | (−4.31) | (−0.95) | (3.40) | (0.06) | (2.00) |
| Quality | 0.1560*** | −0.0958 | 0.0115*** | −0.0096*** | 0.0131 | 0.0184 | 0.0012 | −0.0006** |
| | (4.68) | (−1.02) | (2.72) | (−2.85) | (1.53) | (1.01) | (1.45) | (−2.31) |
| Seasonality | 0.1706*** | −0.2155*** | 0.0128*** | −0.0129*** | −0.0271 | 0.0984*** | −0.0022*** | 0.0025*** |
| | (6.72) | (−2.61) | (3.72) | (−5.04) | (−4.97) | (6.23) | (−3.02) | (4.95) |
| Size | 0.1434*** | −0.1910** | 0.0097*** | −0.0128*** | 0.0065 | −0.0041 | 0.0009 | 0.0029*** |
| | (5.51) | (−2.29) | (2.81) | (−4.93) | (1.18) | (−0.21) | (1.07) | (3.02) |
| Skewness | 0.1535*** | −0.1748** | 0.0109*** | −0.0125*** | −0.0028 | −0.0281*** | 0.0003 | −0.0015*** |
| | (6.03) | (−2.14) | (3.13) | (−4.79) | (−0.85) | (−3.21) | (0.67) | (−2.99) |
| Value | 0.1450*** | −0.1856** | 0.0102*** | −0.0115*** | 0.0013 | 0.0727*** | 0.0002 | 0.0015*** |
| | (5.81) | (−2.52) | (3.12) | (−5.02) | (0.15) | (2.70) | (0.23) | (4.75) |

Panel B. Realized Returns over Months 1–6

| | **All Anomalies** | | | | **Within Cluster** | | | |
|---|---|---|---|---|---|---|---|---|
| **Cluster** | **Mean** | **Variance** | **Skewness** | **Kurtosis** | **Mean** | **Variance** | **Skewness** | **Kurtosis** |
| Accruals | 0.2944*** | −0.2366 | 0.0188** | −0.0205*** | 0.0054 | 0.0410 | −0.0012 | 0.0007 |
| | (6.14) | (−1.55) | (2.56) | (−3.57) | (0.89) | (1.51) | (−0.99) | (0.77) |
| Debt issuance | 0.2970*** | −0.2080 | 0.0169** | −0.0185*** | −0.0087 | 0.0224 | −0.0014 | −0.0021** |
| | (5.61) | (−1.63) | (2.23) | (−3.21) | (−0.65) | (0.90) | (−1.33) | (−2.47) |
| Investment | 0.2671*** | −0.4165*** | 0.0181*** | −0.0221*** | 0.0001 | 0.1265*** | −0.0035** | 0.0008 |
| | (6.32) | (−3.16) | (3.10) | (−5.04) | (0.01) | (4.81) | (−2.42) | (1.29) |
| Low leverage | 0.2719*** | −0.3583*** | 0.0199*** | −0.0218*** | −0.0158 | 0.1460*** | 0.0004 | 0.0034*** |
| | (6.58) | (−3.39) | (3.80) | (−5.63) | (−1.11) | (5.11) | (0.21) | (3.91) |
| Low risk | 0.3374*** | −0.3083*** | 0.0151*** | −0.0241*** | −0.0404* | −0.2276*** | 0.0017 | −0.0009* |
| | (10.54) | (−2.90) | (3.07) | (−5.68) | (−1.70) | (−5.66) | (1.48) | (−1.83) |
| Momentum | 0.2617*** | −0.3477*** | 0.0173*** | −0.0231*** | 0.0422*** | −0.0456 | −0.0036*** | 0.0046*** |
| | (6.70) | (−2.68) | (3.12) | (−5.60) | (3.45) | (−1.28) | (−2.91) | (3.06) |
| Profit growth | 0.2731*** | −0.3501*** | 0.0173*** | −0.0220*** | −0.0016 | 0.0563** | −0.0003 | 0.0042*** |
| | (6.70) | (−2.68) | (2.99) | (−5.23) | (−0.19) | (2.17) | (−0.22) | (4.58) |
| Profitability | 0.2896*** | −0.3837*** | 0.0190*** | −0.0216*** | −0.0067 | 0.0847*** | 0.0001 | 0.0011* |
| | (6.84) | (−2.71) | (3.02) | (−4.75) | (−1.08) | (3.42) | (0.10) | (1.69) |
| Quality | 0.2813*** | −0.2208 | 0.0187** | −0.0181*** | 0.0175 | 0.0263 | 0.0019 | −0.0009** |
| | (5.10) | (−1.40) | (2.51) | (−3.15) | (1.21) | (0.82) | (1.32) | (−2.05) |
| Seasonality | 0.3060*** | −0.4038*** | 0.0215*** | −0.0239*** | −0.0484*** | 0.1445*** | −0.0041*** | 0.0043*** |
| | (7.37) | (−2.99) | (3.71) | (−5.55) | (−5.11) | (5.48) | (−3.23) | (5.11) |
| Size | 0.2575*** | −0.3822*** | 0.0156*** | −0.0240*** | 0.0118 | 0.0088 | 0.0016 | 0.0048*** |
| | (6.13) | (−2.73) | (2.67) | (−5.48) | (1.26) | (0.28) | (1.15) | (3.07) |
| Skewness | 0.2860*** | −0.3476*** | 0.0191*** | −0.0232*** | −0.0142*** | −0.0394*** | −0.0003 | −0.0019** |
| | (6.89) | (−2.58) | (3.27) | (−5.34) | (−2.98) | (−2.85) | (−0.51) | (−2.41) |
| Value | 0.2502*** | −0.3664*** | 0.0171*** | −0.0214*** | 0.0095 | 0.1601*** | 0.0001 | 0.0026*** |
| | (5.98) | (−2.99) | (3.05) | (−5.60) | (0.62) | (3.48) | (0.05) | (5.51) |

Panel C. Realized Returns over Months 1–12

| | **All Anomalies** | | | | **Within Cluster** | | | |
|---|---|---|---|---|---|---|---|---|
| **Cluster** | **Mean** | **Variance** | **Skewness** | **Kurtosis** | **Mean** | **Variance** | **Skewness** | **Kurtosis** |
| Accruals | 0.1802*** | −0.3476** | 0.0015 | −0.0207*** | 0.0109 | 0.0373* | −0.0008 | 0.0012 |
| | (3.67) | (−2.19) | (0.21) | (−3.65) | (1.61) | (1.78) | (−0.64) | (1.12) |
| Debt issuance | 0.1965*** | −0.2926** | 0.0003 | −0.0198*** | −0.0059 | −0.0038 | −0.0004 | −0.0018* |
| | (3.57) | (−2.19) | (0.04) | (−3.44) | (−0.42) | (−0.13) | (−0.37) | (−1.85) |
| Investment | 0.1920*** | −0.4075*** | 0.0042 | −0.0222*** | 0.0032 | 0.0836*** | −0.0027 | 0.0016** |
| | (4.44) | (−3.00) | (0.70) | (−5.18) | (0.31) | (3.31) | (−1.73) | (2.45) |
| Low leverage | 0.1955*** | −0.3574*** | 0.0054 | −0.0222*** | −0.0161 | 0.1268*** | 0.0031 | 0.0031*** |
| | (4.62) | (−3.18) | (0.97) | (−5.80) | (−1.13) | (3.92) | (1.64) | (3.18) |
| Low risk | 0.2510*** | −0.3716*** | 0.0042 | −0.0245*** | −0.0339 | −0.1305*** | 0.0003 | −0.0008 |
| | (8.20) | (−3.23) | (0.75) | (−5.69) | (−1.42) | (−3.04) | (0.23) | (−1.64) |
| Momentum | 0.1996*** | −0.3066** | 0.0074 | −0.0233*** | 0.0114 | −0.1650**** | 0.0038*** | 0.0054*** |
| | (4.97) | (−2.36) | (1.32) | (−5.72) | (1.04) | (−4.50) | (3.14) | (3.48) |
| Profit growth | 0.2023*** | −0.3566*** | 0.0042 | −0.0226*** | −0.0220*** | 0.0344 | 0.0008 | 0.0054*** |
| | (4.87) | (−2.57) | (0.69) | (−5.39) | (−2.82) | (1.48) | (0.56) | (5.70) |
| Profitability | 0.2105*** | −0.4035*** | 0.0036 | −0.0236*** | −0.0146** | 0.0601** | 0.0001 | 0.0009 |
| | (4.99) | (−2.74) | (0.58) | (−5.48) | (−2.09) | (2.56) | (0.08) | (1.27) |
| Quality | 0.2217*** | −0.3212* | 0.0053 | −0.0203*** | −0.0183 | 0.0232 | −0.0016 | −0.0010* |
| | (3.96) | (−1.95) | (0.70) | (−3.43) | (−1.30) | (0.70) | (−1.13) | (−1.87) |
| Seasonality | 0.2050*** | −0.4079*** | 0.0047 | −0.0246*** | −0.0150 | 0.0909*** | 0.0000 | 0.0041*** |
| | (4.72) | (−2.88) | (0.75) | (−5.76) | (−1.41) | (3.05) | (0.02) | (4.74) |
| Size | 0.1573*** | −0.4481*** | −0.0020 | −0.0249*** | 0.0272*** | 0.0708** | 0.0047*** | 0.0044*** |
| | (3.70) | (−3.13) | (−0.35) | (−5.75) | (2.81) | (2.27) | (3.06) | (3.15) |
| Skewness | 0.2206*** | −0.3751*** | 0.0065 | −0.0243*** | −0.0305*** | −0.0136 | −0.0018*** | 0.0010 |
| | (5.26) | (−2.66) | (1.09) | (−5.58) | (−6.43) | (−0.82) | (−2.78) | (1.20) |
| Value | 0.1300*** | −0.3361*** | 0.0055 | −0.0203*** | 0.0352** | 0.2067*** | −0.0033** | 0.0007 |
| | (3.28) | (−2.62) | (0.95) | (−5.38) | (2.40) | (4.51) | (−2.34) | (1.35) |