Enhancing Stock Return Prediction and Portfolio Performance with Adaptive LASSO

Abstract: This paper explores the predictability of monthly US stock returns using adaptive LASSO on firm-specific characteristics from June 1990 to December 2022. By efficiently selecting relevant features and managing high-dimensional data, adaptive LASSO improves return forecasts over traditional models. Key predictors include lagged returns, mean log-volumes, market values, dividend yields, and R&D expenses. We design two threshold-based portfolios: Adaptive LASSO 1/N (equal-weighted) and Adaptive LASSO SR (Sharpe ratio-weighted), incorporating a 0.3% transaction cost and a no-trade region to reduce turnover. These portfolios are evaluated against two benchmarks: the equal-weighted portfolio and the S&P 500 index. Both threshold-based portfolios outperform the benchmarks, with the Adaptive LASSO SR portfolio showing the best performance, while demonstrating resilience to transaction costs.

Keywords: Adaptive LASSO, Stock Return Forecasting, Portfolio Optimization, Cross-Section Features

JEL codes: G11; G17; C53; C63

1. Introduction

Forecasting stock returns and constructing optimal portfolios remain fundamental challenges in finance. Traditional approaches, such as the mean-variance framework, seek to optimize returns for a given level of risk but often rely on estimations that are highly sensitive to errors. These inaccuracies can lead to significant underperformance in real-world applications.

Over the past decades, extensive research has focused on the link between firm-specific characteristics and stock returns. Studies have emphasized the role of financial ratios, momentum, and other factors in explaining cross-sectional return predictability. Despite these advances, accurately forecasting stock returns remains an elusive goal due to the complex and dynamic nature of financial markets, as well as the high-dimensional nature of the predictor space.

Recent advancements in machine learning, particularly in the use of regularization techniques, have enhanced forecasting accuracy by addressing the challenges of feature selection and high dimensionality, while also reducing the risk of overfitting. Adaptive LASSO (Least Absolute Shrinkage and Selection Operator) is one such technique. Adaptive LASSO refines the original LASSO methodology by enabling more accurate identification of relevant predictors, especially when they are highly correlated. This improved version asymptotically identifies true nonzero coefficients and enhances predictive performance by managing multicollinearity and overfitting, common issues in stock return predictions.

While predictive accuracy is crucial, successful portfolio management also depends on transaction costs and turnover rates. High turnover strategies, such as those used in high-frequency trading, can incur substantial transaction costs, which may erode net returns. Several papers have highlighted the importance of incorporating transaction costs into portfolio optimization.

This paper examines the predictability of monthly U.S. stock returns using adaptive LASSO on a dataset spanning from June 1990 to December 2022. By leveraging lagged firm-specific characteristics, the adaptive LASSO model forecasts cross-sectional stock returns, adjusting to time-varying predictor spaces.

The empirical results of this study indicate that no single predictor consistently dominates in return forecasting, but variables related to size, momentum, and investment activities show high selection frequencies, reflecting their relevance in stock return prediction. Building on these forecasts, two trading strategies based on adaptive LASSO are developed: one with equal weights (Adaptive LASSO 1/N) and the other weighted by expected Sharpe ratios (Adaptive LASSO SR). These strategies are compared against a traditional 1/N portfolio and the S&P 500 index. The results show that adaptive LASSO portfolios consistently outperform the benchmarks in terms of cumulative net log-returns, Sharpe ratios, and Sortino ratios, even after accounting for transaction costs. This demonstrates the potential of adaptive LASSO to enhance portfolio performance while considering real-world constraints such as transaction costs and turnover rates.

The contribution of this paper is threefold: (1) applying adaptive LASSO to a long and updated sample of U.S. stock returns to assess the predictive power of firm features, (2) developing trading strategies that incorporate transaction costs and no-trade zones, and (3) comparing these strategies against established benchmarks using a comprehensive set of financial performance metrics.

The rest of the paper is organized as follows: Section 2 reviews the related literature, Section 3 describes the dataset and predictor variables used in the study, Section 4 outlines the forecasting methodology and portfolio construction techniques, Section 5 presents the empirical results, and Section 6 concludes the paper.

2. Literature review

Fundamental stock features have proven to be valuable indicators for building stock portfolios. Abarbanell and Bushee (1998) show that portfolios based on several fundamental indicators have a high cumulative size-adjusted abnormal return, with a significant portion of these abnormal returns generated around earnings announcements. Piotroski (2000) proposes an aggregate FSCORE, based on nine fundamental signals of firm profitability, liquidity/leverage and operational efficiency to classify firms, and shows that high FSCORE stocks have higher profitability. Turtle and Wang (2017), Tikkanen and Äijö (2018) and Walkshäusl (2020) revisited the work of Piotroski (2000) and concluded that high-FSCORE firms significantly outperform low-FSCORE firms in several international stock markets. Green et al. (2017) add 94 accounting features to the Fama-French-Carhart five-factor model and show that 12 characteristics provide relevant information about the average US monthly returns of non-microcap stocks over the full period 1980-2014. Dechow et al. (2001) look at firms with low fundamentals ratios and show that these firms provide short sellers with profitable investment strategies. Yan and

Zheng (2017) use more than 18,000 fundamental signals, showing that many of these signals are significant predictors even after accounting for data mining. Most notably, the seminal paper of Brand et al. (2009) provides a simple, parsimonious, flexible and effective framework for directly modelling the portfolio weights as a function of an asset's characteristics. With just three superimposed characteristics (size, book-to-market ratio, and momentum), Brandt et al. (2009) found a robust performance in- and out-of-sample.

Another stream of literature uses technical indicators to predict future stock returns, with mixed results. Park and Irwin (2007) and Nazário et al. (2017) are two survey papers on technical analysis applied to stocks. More recently, the trend has been to combine fundamental and technical indicators (see, for instance, Yuan et al., 2020; Li et al., 2022).

Given the richness of data available, there is a bourgeoning literature proposing new procedures, most of which use machine learning tools to reduce the dimensionality of the predictor space. For instance, Kozak et al. (2020) develop a shrinkage factor that provides effective feature selection and produces highly significant abnormal returns relative to the five-factor model. Gu et al. (2020) evaluate the effectiveness of various machine learning models in predicting stock returns using a dataset of nearly 30,000 stocks across 60 years. They find that tree-based models and neural networks deliver the best performance, particularly when using price trends, liquidity, and volatility as predictors. Other examples of machine learning applied to portfolio optimization are Almahdi and Yang (2017), Kircher and Rösch (2021), Liu et al. (2020), and Zhang et al. (2020).

LASSO techniques have gained popularity due to their ability to eliminate redundant variables, enhancing prediction accuracy and portfolio performance. Adaptive LASSO, introduced by Zou (2006), modifies traditional LASSO by performing a two-stage shrinkage, allowing consistent variable selection even under relaxed conditions. Messmer and Audrino (2017) demonstrated that adaptive LASSO outperformed traditional LASSO in predicting US stock returns, selecting 14 relevant features, namely short-term reversal, twelve-month momentum, and research spending scaled by market value. Other extensions of LASSO have been explored to improve model performance, such as the adaptive group LASSO of Freyberger et al. (2020), the twin adaptive LASSO of Lee et al. (2022) and applications of LASSO in quantile regressions (Bonaccolto et al., 2018; Fan et al., 2023).

Beyond the importance of inputs in portfolio optimization and methods for addressing high-dimensionality challenges, the literature also provides insights into additional issues faced by investors in real markets. Fundamentally, a risk-averse investor seeks to maximize the reward-to-risk ratio, for which the Shape ratio is the obvious metric, after accounting for transaction costs (Hung et al., 2000).

Transaction costs play a critical role in accurate portfolio evaluation. They are incorporated into portfolio optimization either by directly including them in the objective function or by imposing constraints on individual or total turnovers and portfolio weights. DeMiguel et al. (2020) incorporate proportional transaction costs into the parametric portfolio policies of Brandt et al. (2009) with a LASSO constraint and a transaction trigger threshold. Lee and Yoo (2020) applied this concept using long short-term memory networks, showing that thresholds between 0 and 0.025 improve returns and reduce risk.

In summary, the evolving landscape of portfolio optimization and stock return prediction reflects significant data-driven advancements, with cross-sectional fundamental features proven effective for return forecasting, complemented by technical indicators for enhanced predictive accuracy. The integration of machine learning models, particularly adaptive LASSO, has addressed the challenges of high-dimensional data and multicollinearity, leading to improved feature selection and return predictability. Additionally, the consideration of transaction costs in threshold-based portfolios provides more realistic frameworks. These advancements underscore the importance of adaptive models applied to high-dimensional data for optimizing portfolio performance.

3. Data

This study analyses the 500 largest stocks by market capitalization traded on the NYSE or NASDAQ, using 51 stock market and accounting features from Refinitiv Eikon. The dataset has a daily frequency and covers the period from January 1, 1990, to December 31, 2022. We have also obtained the S&P 500 Total Return Index from the same source. This index considers the reinvestment of dividends and accommodates price changes resulting from management events (e.g., stock splits). Missing data is filled with the most recent available value. To create a monthly dataset, we use Wednesday data to represent four-week months, yielding a total of 430 monthly observations. For the risk-free rate, we use the annualized daily 3-month T-bill secondary market yield from FRED (Federal Reserve Bank of St. Louis), as the 1-month rate was unavailable for the entire period. These rates are converted to monthly values assuming 28-day months, hence resulting in 13 months per year.

Over the 33-year period, some firms exited the database due to mergers (140 cases), acquisitions (246 cases), split-offs (5 cases), name changes (5 cases), bankruptcies (6 cases), privatizations (13 cases), and other events. To maintain a constant number of stocks, each departing firm was replaced using the following rules: (1) For mergers or acquisitions, the departing stock was replaced by the merging entity or acquiring firm's stock if it was not already in the database on the last trading day of the replaced stock, (2) in split-offs, the successor company with the highest market value replaced the original firm, (3) for name changes, the stock remained in the database under its new name, and (4) in all other cases, the firm was replaced by the stock with the highest market capitalization not yet in the database. Data for the new stocks were included from the first day following the exit of the replaced firm. In total, the study used information from 931 stocks throughout the period.

After collecting the data, we perform several transformations to prepare it for estimation. Most of the accounting features have high granularity, with a yearly periodicity. To increase the comparability of these features across firms, in some cases, we use the market value as a deflator. We calculate the arithmetic monthly returns for each stock and for the S&P 500 index using the total return index, and we use the signed Amihud illiquidity ratio and signed Parkinson volatility measure.

Liquidity, as defined by Amihud and Mendelson (1986), refers to the ability to trade an asset quickly without significantly affecting its price. Amihud's illiquidity ratio (Amihud, 2002) measures the price impact per dollar traded. We calculate the monthly illiquidity ratio, adjusted by the sign of the monthly returns, as follows:

$$ILLIQ_{i,t} = \frac{Sign(R_{i,t})}{M} \sum_{\tau=1}^{M} \frac{|R_{i,\tau}|}{V_{i,\tau}}, \tag{1}$$

where $R_{i,\tau}$ and $V_{i,\tau}$ are the arithmetic return and trading volume, in USD, of stock i on the τ-th day of month t, respectively. M is the number of trading days in month t, and $Sign(R_{i,t})$ takes the value -1 if the return in month t is negative and 1 otherwise.

Volatility is measured by the Parkinson (1980) estimator, also corrected by the sign of the monthly returns:

$$\sigma_{i,t} = \frac{Sign(R_{i,t})}{\sqrt{M}\sqrt{4ln(2)}}\sqrt{\sum_{\tau=1}^{M}\left(ln\left(\frac{H_{i,\tau}}{L_{i,\tau}}\right)\right)^2}, \quad (2)$$

where $H_{i,\tau}$ and $L_{i,\tau}$ are the high and low prices of stock i on the τ-th day of month t, respectively.

Before proceeding with the empirical analysis, we pre-filtered the predictor space by excluding some features with high levels of multicollinearity, measured by the Variance Inflation Factor (VIF). This procedure eliminated several features, leaving 39 remaining.

Table A.1 in the appendix shows the 39 features and their description according to the notes available in the Refinitiv Eikon database.

4. Methodology

4.1. Forecasting

We forecast the monthly returns on US stocks using lagged firm features, with 1 to 4 lags for arithmetic returns and 1 lag for other predictors. To standardize explanatory variables of varying magnitudes, we scaled them to align with the mean scale of returns.

When a stock is replaced in the database, its substitute is not included in the regression and in the portfolios for 5 months, to ensure that lagged predictors are available. So, the monthly regressions may have a different number of stocks, N_t. To handle the remaining missing data, we create dummy variables that equal 1 when a feature is missing and 0 otherwise, replacing missing values with 0. Linearly dependent dummies are removed before adding them to the predictor set.

The original LASSO estimator, proposed by Tibshirani (1996), is defined as follows:

$$\widehat{\boldsymbol{\beta}}_{LASSO,t} = \underset{\boldsymbol{\beta}}{argmin}\left\{\sum_{i=1}^{N_t}\left(R_{i,t} - \beta_0 - \sum_{j}\beta_{j,t}x_{i,j,t}\right)^2 + \lambda_t\sum_{j}\beta_{j,t}^2\right\}, \lambda_t \geq 0, \quad (3)$$

where $R_{i,t}$ is the discrete return for firm i in month t and $x_{i,j,t}$ is the explanatory variable j for firm i in month t. The estimation is performed for $t = 1, \ldots, T$, where $T = 425$ is the sample size. $\beta_{j,t}$ is the regression coefficient for the explanatory variable j in month t, and β_0 is the constant term. Finally, λ_t is the regularisation (or penalty) parameter in month t, which allows for the elimination of redundant coefficients.

Adaptive LASSO, proposed by Zou (2006), differs from standard LASSO by allowing the weights to vary for each parameter, defining a first-stage estimator so that relevant variables are less affected by the penalty, and allowing consistent variable selection by the procedure in the second-stage estimator. The adaptive LASSO estimator is given by:

$$\widehat{\boldsymbol{\beta}}_{Ad-LASSO_t} = \underset{\boldsymbol{\beta}}{argmin} \left\{ \sum_{i=1}^{N_t} \left(R_{i,t} - \beta_0 - \sum_j \beta_{j,t} x_{i,j,t} \right)^2 + \lambda_t \sum_j \widehat{\omega}_{j,t} \left| \beta_{j,t} \right| \right\}, \lambda_t \geq 0, \qquad (4)$$

where $\widehat{\omega}_{j,t}$ is a weight vector. Following the suggestion of Zou (2006) we define $\widehat{\omega}_{j,t} = 1/\left|\hat{\beta}_{j,t}^{OLS}\right|^{\gamma_t}$, where $\hat{\beta}_{j,t}^{OLS}$ is the OLS first stage estimator and $\gamma > 0$. While Zou suggests using $\gamma = 1$, in this paper, we dynamically select this parameter for each month from the set $\gamma = \{0.5, 1, 1.5, 2\}$ via a 10-fold cross-validation procedure.

4.2. Trading strategies

For comparison, we first examine the 1/N portfolio, also termed the naïve strategy, which avoids the risks of parameter estimation errors found in traditional models. DeMiguel et al. (2009) compared the 1/N portfolio to 14 other models and found that none of these models outperformed the naïve strategy after accounting for transaction costs. They attributed this to the impact of estimation errors in reducing the benefits of optimal diversification. Consequently, the 1/N portfolio has been widely adopted as a benchmark for evaluating more sophisticated trading strategies and as a baseline portfolio for other strategies (Tu and Zhou, 2011; Lee and Yoo, 2020; Jiang et al., 2019).

In addition, we take the S&P 500 index as a second benchmark. The main advantage of this benchmark is that it represents a buy-and-hold strategy with no transaction costs for a well-diversified market-weighted portfolio.

We base stock selection on the predicted monthly returns for each stock, accounting for transaction costs. In this study, we consider proportional round-trip transaction costs of 0.3%. Earlier papers, such Brandt et al. (2009) and DeMiguel et al.

(2009) consider proportional costs of 0.5%. However, more recent papers consider lower transaction costs (see, for instance, Babazadeh and Esfahanipour, 2019; Paiva et al., 2019; Guo et al., 2021). A round-trip cost of 0.3% seems reasonable for liquid markets such as NYSE and NASDAQ.

The portfolios are rebalanced monthly in line with the periodicity of the data. We assume that short selling is not allowed, so all the stocks have non-negative weights in the portfolio. To minimize trades that become unprofitable after transaction costs, a long position in a stock is initiated in month t only if its expected return in the following month $(t+1)$ exceeds the cost threshold of 0.15%. Conversely, a long position is closed in month t if the expected return is lower than -0.15%, i.e., when the expected loss exceeds the cost of exiting the market. In all other cases, the investor maintains the position in the stock from the previous month.

After determining which stocks to include in the portfolio, it is necessary to compute their weights. We consider two alternatives: an equal-weighted portfolio with the selected stocks and a portfolio with weights based on the expected Sharpe ratio of each selected stock. The first case, referred to as Adaptive LASSO 1/N, assigns an equal weight to each stock i at month t, defined as $w_{i,t}^{*} = \frac{1}{S_t}$, where S_t is the number of stocks for the portfolio in month t. The alternative approach, hereafter referred to as Adaptive LASSO SR, is based on the expected adjusted Sharpe ratios (Israelsen, 2005), such that

$$E\left(\widehat{SR}_{i,t+1}\right) = \frac{E\left(R_{i,t+1}\right) - R_{f,t}}{\left(\sigma_{i,t}^{\tilde{R}}\right)^{Sign\left(E\left(R_{i,t+1}\right) - R_{f,t}\right)}}, \tag{5}$$

where $E\left(R_{i,t+1}\right)$ is the expected return of stock i in month $t+1$, $R_{f,t}$ is the risk-free return in month t, $\sigma_{i,t}^{\tilde{R}}$ is the standard deviation of the excess daily returns for stock i over the previous 4 months (80 days) and $Sign(x)$ is a function that takes the value -1 if x is negative and 1 otherwise.

We define the weight for each stock as the difference between its expected adjusted Sharpe ratio and the minimum expected adjusted Sharpe ratio across all stocks in the portfolio, $E\left(\widehat{SR}_{min,t+1}\right)$. This difference is then normalized by dividing it by the sum of such differences across the portfolio:

$$w_{i,t+1}^{*} = \frac{E(\widehat{SR}_{i,t+1}) - E(\widehat{SR}_{min,t+1})}{\sum_{i=1}^{S_t}[E(\widehat{SR}_{i,t+1}) - E(\widehat{SR}_{min,t+1})]}. \quad (6)$$

After determining these weights, which we will also refer to as the target weights, we consider a non-trade radius similar to the one used by Brandt et al. (2009), which is based on prior research (Magill and Constantinides, 1976; Taksar et al., 1988; Davis and Norman, 1990; Leland, 2000).

First, a "hold" portfolio is defined, assuming no trades. This way, the weights of this portfolio change between periods $t-1$ and t only due to the price changes of the assets. The weights of the hold portfolio, $w_{i,t}^{h}$, are expressed as:

$$w_{i,t}^{h} = w_{i,t-1}\frac{1+R_{i,t}}{1+R_{p,t}}, \quad (7)$$

where $w_{i,t-1}$ is the weight of stock i at time $t-1$, $R_{i,t}$ is the arithmetic return of stock i at time $t-1$ and $R_{p,t}$ is the arithmetic return of the portfolio at time t. The actual portfolio weights depend on the target weights and the "hold" portfolio weights. They are defined as:

$$w_{i,t} = \begin{cases} w_{i,t}^{h} & , if\ \frac{1}{N_t}\sum_{i=1}^{S_t}\left(w_{i,t}^{*} - w_{i,t}^{h}\right)^2 \leq k^2 \\ \alpha_t w_{i,t}^{h} + (1-\alpha_t)w_{i,t}^{*}, & if\ \frac{1}{N_t}\sum_{i=1}^{S_t}\left(w_{i,t}^{*} - w_{i,t}^{h}\right)^2 > k^2 \end{cases}, \quad (8)$$

where S_t is the number of stocks in the portfolio at time t, α_t is the time-varying linear combination parameter and k is the non-trade radius.

The no-trade region is a hypersphere of radius k around the target portfolio weights. So, if the hold portfolio is sufficiently close to the target portfolio, it is better not to trade, and if the hold portfolio is sufficiently far from the target, the trade must occur to the frontier of the no-trade region, the new portfolio being a weighted average of the hold and target portfolios. The parameter α_t can be chosen so that the new portfolio is exactly at the boundary, that is:

$$\frac{1}{S_t}\sum_{i=1}^{S_t}\left(w_{i,t}^{*} - w_{i,t}^{h}\right)^2 = \frac{1}{S_t}\left(w_{i,t}^{*} - \alpha_t w_{i,t}^{h} - (1-\alpha_t)w_{i,t}^{*}\right)^2 = \alpha_t^2\sum_{i=1}^{S_t}\left(w_{i,t}^{*} - w_{i,t}^{h}\right)^2. \quad (9)$$

Setting $\left(w_{i,t}^{*} - \alpha_t w_{i,t}^{h} - (1-\alpha_t)w_{i,t}^{*}\right)^2 = k^2$, then

$$\alpha_t = \frac{k\sqrt{S_t}}{\left(\sum_{i=1}^{S_t}\left(w_{i,t}^* - w_{i,t}^h\right)^2\right)^{1/2}}. \quad (10)$$

For the selection of the hyperparameter k, we split the data into in-sample and out-of-sample periods. The in-sample covers the first 170 months, approximately 2/5 of the entire sample period, and the strategies are implemented during this period using values for k from 0 to 0.01, with a step size of 0.001. We then use the value of k that maximizes the annualized Sharpe ratio in-sample. After selecting k, we build and evaluate the portfolios over the test period, which covers the remaining months. Note that the no-trade region is not considered for the benchmark strategies.

5. Empirical results

5.1. Forecasting

Firstly, we analyse the relevance of the features for forecasting expected returns. Table A.2 in the appendix reports the percentage of the months in the overall period (including in-sample and out-of-sample periods) in which each feature is selected by the adaptive LASSO for forecasting the returns.

The mean of the four previous values of log-volumes is the most consistently chosen feature, with an overall selection frequency of 46.35%. The natural logarithms of the volume, the market value, and the number of both full and part-time employees also present themselves as good predictors, with selection frequencies of 39.77%, 38.59% and 32.24%, respectively. These results highlight the importance of size proxies in predicting returns, as pointed out by Freyberger et al. (2020). Lagged returns of the previous one to four months are chosen more than 25% of the time as predictors, implying the existence of information in momentum. The importance of momentum has long been referred to in the literature (see, for instance, Carhart,1997; Fama and French, 2016; Brandt et al., 2009; Freyberger et al., 2020). Additionally, the relative importance of Research and Development (R&D) (32.47% frequency) and dividend yield (31.77% frequency) underscore the relevance of R&D expenditures (Messmer and Audrino, 2017) and dividend yields (Lettau and Ludvigson, 2005; Cochrane, 2008).

There are notable differences between in-sample and out-of-sample selection frequencies. For instance, the natural logarithm of the volume is chosen in 31.18% of in-sample months but in 45.49% of out-of-sample months, suggesting a shift in the importance of trading volume over time. Conversely, the turnover rate drops sharply from 30.00% in-sample to 4.31% out-of-sample, indicating instability in its predictive power. This suggests the presence of return continuation or reversal patterns in the market. Adaptive LASSO effectively adjusts to evolving market conditions, supporting its use for dynamic stock return forecasting.

Table 1 shows some descriptive statistics of the percentage of months in which the features are selected by adaptive LASSO. Results concerning the individual features are presented in Table A2 in the Appendix.

Table 1 – Descriptive statistics of the relative number of months of feature selection

	Overall sample	In-sample	Out-of-sample
Mean	24.81	24.99	24.69
Median	24.24	25.00	24.91
Std. Dev	7.46	7.07	8.97
Minimum	9.88	10.59	4.31
Percentile 5	14.64	14.80	12,20
Percentile 95	38.28	35.29	41.53
Maximum	46.35	40.00	50.59
Nº features >25%	20	21	21

Notes: This table presents some descriptive statistics of the relative number of months in which features were selected by the adaptive LASSO. These statistics were computed using the information presented in Table A.2. The last row refers to the number of features chosen in at least 25% of the months. All values are in percentage except the last row.

The mean selection frequency is consistent across all periods, with 24.81% overall, 24.99% in-sample, and 24.69% out-of-sample, indicating a balanced selection process without significant bias. The standard deviation is lowest in-sample (7.07%) and highest out-of-sample (8.97%), reflecting greater variability in feature selection during the out-of-sample period, which could be due to changing market dynamics or structural breaks. The minimum selection frequency is particularly low out-of-sample (4.31%). Conversely, the maximum frequency is highest out-of-sample (50.59%), indicating that certain features were consistently relevant. The relatively stable means and medians suggest that adaptive LASSO maintains a balanced selection process across both periods. The number of features selected in at least 25% of the months remains consistent, with 20 overall, 21 in-sample, and 21 out-of-sample. These findings illustrate the Adaptive LASSO's flexibility in responding to evolving market dynamics, underscoring its predictive power and reliability for stock return forecasting.

5.2. Performance of the portfolios

Table 2 presents key performance indicators of the cumulative returns and other portfolio evaluation metrics out-of-sample.

Table 2 - Performance with transaction costs

	S&P 500	1/N	Adaptive LASSO 1/N	Adaptive LASSO SR
Win rate	68.24	68.24	66.27	64.71
CumRet	189.57	213.24	269.22	337.20
Total turnover		130.97	12,978.49	14,520.23
Turnover reduction			49.48	53.72
Skewness	-0.4755	-0.3405	0.5064**	0.5953***
Annualized mean return	11.12	12.56	15.70	20.11**
Annualized standard deviation	16.44	17.73	19.96	24.59
SR	60.18	63.90	72.41*	76.74**
SOR	129.57	143.80	214.74*	255.34**
CE ($\gamma = 1$)	9.66	10.87	13.72	17.19**
CE ($\gamma = 3$)	6.69	7.47	10.41	12.45*
CE ($\gamma = 5$)	2.64	2.70	5.82	5.82
$CVaR_{1\%}$	21.48	22.85	18.16	21.41
$CVaR_{5\%}$	12.03	12.07	10.09**	11.72
MD	52.15	50.03	36.01	42.16

Notes: This table presents the performance metrics of the two proposed trading strategies (Adaptative LASSO 1/N and Adaptative LASSO SR) and the S&P 500 and 1/N benchmark portfolios in the presence of round-trip transaction costs of 0.3%. These metrics are computed out of sample. $T = T_1 + T_2$ is the size of the whole sample and $T_1 = 170$ and $T_2 = 255$ are the sizes of the in-sample and out-of-sample periods, respectively. The win rate corresponds to the percentage of months in the market in which the strategy generates a positive return. The cumulative return after transaction costs of strategy p is presented as CumRet, defined as $\text{CumRet} = \prod_{t=T_1+1}^{T}(1 + R_{p,t}) - 1$. The total turnover is the sum of the turnover for all the periods, with turnover being defined as the sum of the absolute changes in portfolio weights from one period to the next, $Turnover_p = \sum_{t=T_1+1}^{T}\sum_{i=1}^{S_t}|w_{i,t} - w_{i,t-1}|$. The turnover reduction is the one generated by the implementation of the no-trade radius. The annualized mean return and standard deviation are the average return and the standard deviation of the monthly returns multiplied by 13 and $\sqrt{13}$, respectively. We consider 4-week periods, so we have 13 periods per year. The annualized Sharpe ratio (SR) corresponds to the ratio between the annualized mean return minus the annualized average free-risk rate (proxied by the 3-month T-bill secondary market yield) and the annualized standard deviation of excess returns. The annualized Sortino ratio considers in the denominator only the negative deviations from a target value, which is assumed to be zero, $SOR_p = \sqrt{13}E(R_p)/\sqrt{\frac{1}{T_2}\sum_{t=T_1+1}^{T} min\left[R_{p,t}, 0\right]^2}$. The annualized certainty equivalent (CE) reflects the indifference of the investor between obtaining a certain amount or investing and in the risky strategy. Assuming a Constante Relative Risk Aversion (CRRA) utility function $CE = \frac{13}{T_2}\sum_{t=T_1+1}^{T} ln\,(1 + R_t)$ and $CE = \left(\frac{1}{T_2}\sum_{t=T_1+1}^{T}(1 + R_t)^{1-\gamma}\right)^{\frac{13}{1-\gamma}} - 1$ if the risk-aversion parameter γ is equal to 1 or higher than 1, respectively. CVaR at $\alpha\%$ (Conditional Value-at-Risk at $\alpha\%$) measures the average loss conditional on the VaR (Value-at-Risk) at the $\alpha\%$ level being exceeded and the maximum drawdown (MD) is computed as the maximum observed loss from a peak to a trough of the accumulated value of the trading strategy, as a percentage of the value of that peak. The inference is obtained using bootstrap p-values, which give the probability that a certain metric is worse than that of the best benchmark strategy, 1/N. These p-values are obtained using 100,000 bootstrap samples created with the circular block procedure of Politis and Romano (1994), with an optimal block size chosen according to Politis and White (2004) and Politis and White (2009). The significance of the metrics at the 10%, 5% and 1% levels are denoted by "*", "**" and "***", respectively. All values are in percentage except skewness.

As expected, the results show that the total turnover is substantially lower for the 1/N portfolio (approximately 131%), whereas the Adaptive LASSO 1/N and Adaptive LASSO SR portfolios exhibit significantly higher turnover values, by factors of approximately 100 and 110, respectively. However, the implementation of the no-trade boundary proposed by Brandt et al. (2009) led to considerable reductions in turnover of 49.48% and 53.72%, respectively. This reduction highlights the effectiveness of transaction cost mitigation techniques in improving portfolio efficiency.

In terms of return distribution characteristics, the benchmark portfolios (S&P 500 and 1/N) display left-skewed monthly returns, indicating a higher frequency of negative outliers, whereas the Adaptive LASSO portfolios exhibit right-skewed returns, suggesting a tendency for occasional higher-than-average positive returns. The win rate exceeds 64% across all portfolios, with the S&P 500 and 1/N benchmarks achieving slightly superior win rates of 68.24% compared to the Adaptive LASSO portfolios.

Regarding performance, the Adaptive LASSO SR portfolio exhibits the highest annualized mean return (20.11%), followed by the Adaptive LASSO 1/N portfolio (15.70%), with both outperforming the S&P 500 (11.12%) and 1/N (12.56%) benchmarks. However, this superior return performance comes at the cost of increased risk, as reflected in the higher annualized standard deviation for the Adaptive LASSO SR portfolio (24.59%) and the Adaptive LASSO 1/N portfolio (19.96%). Despite the higher volatility, the Adaptive LASSO SR portfolio has the highest annualized Sharpe ratio (76.74%), followed by the Adaptive LASSO 1/N portfolio (72.41%), outperforming the S&P 500 (60.18%) and 1/N (63.90%). This indicates that, on a risk-adjusted basis, the Adaptive LASSO strategies generate higher returns per unit of risk, although the statistical significance of these improvements is limited.

A more pronounced difference is observed in the annualized Sortino ratio, which accounts for downside risk. The Adaptive LASSO SR and Adaptive LASSO 1/N portfolios achieve Sortino ratios of 255.34% and 214.74%, respectively, which are significantly higher than those of the S&P 500 (129.57%) and 1/N (143.80%) portfolios. These findings suggest that the Adaptive LASSO-based strategies provide more favorable downside risk-adjusted returns, enhancing their attractiveness to risk-averse investors.

The Adaptive LASSO SR portfolio further stands out in terms of the annualized certainty equivalent, suggesting higher investor utility relative to other strategies. Similarly, the Adaptive LASSO 1/N portfolio outperforms the benchmark portfolios in this metric.

In terms of downside risk measures, the Conditional Value-at-Risk (CVaR) shows values above 18% at the 1% level and above 10% at the 5% level. This suggests a substantial risk of loss in extreme market conditions. Among all portfolios, the Adaptive LASSO 1/N portfolio presents the lowest CVaR values, suggesting a relatively lower exposure to severe tail risk events.

Lastly, the maximum drawdown, which captures the most significant peak-to-trough decline, remains above 35% for all portfolios. The highest drawdown is observed in the S&P 500 strategy (52.15%), indicating substantial vulnerability to market downturns. Conversely, the Adaptive LASSO 1/N portfolio records the lowest drawdown (36.01%), reflecting greater resilience to extreme losses.

Overall, the results highlight the superior performance of the Adaptive LASSO SR strategy, which consistently outperforms the benchmark portfolios. Even after incorporating transaction costs, the Adaptive LASSO-based strategies demonstrate strong risk-adjusted returns and robustness in extreme market conditions. The combination of threshold-based selection with a no-trade boundary proves effective in optimizing portfolio performance while mitigating transaction costs. These findings underscore the potential of Sharpe ratio-based weight optimization in delivering superior investment outcomes.

5.3. Sensitivity of the results to transaction costs

The evolution of the cumulative returns of the LASSO-base strategies and cumulative returns of the benchmarks, with and without transaction costs, are shown in Figure 1. The key feature observed in this figure is that both Adaptive LASSO portfolios consistently outperform the 1/N benchmark, with the SR variant achieving the highest cumulative return of 337.2% with transaction costs and 356.66% without transaction costs, significantly surpassing all other portfolios.

Figure 1 – Cumulative returns with and without transaction costs

a) With transaction costs (c = 0.15%) b) Without transaction costs

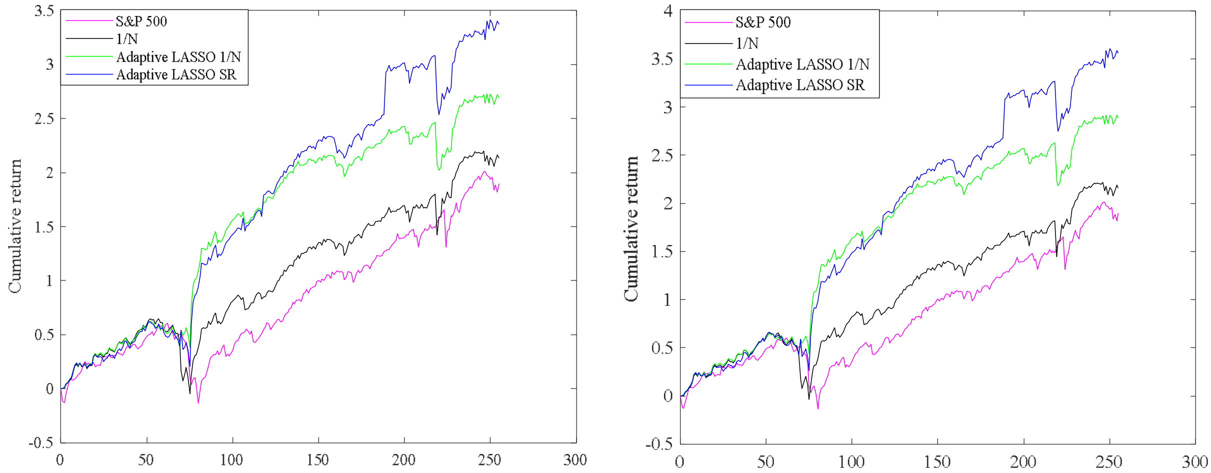

Table 3 presents the performance metrics of the portfolios in the scenario with no transaction costs. The comparison between Table 2 (with transaction costs) and Table 3 (without transaction costs) highlights the impact of transaction costs on portfolio performance.

Without transaction costs, cumulative returns are noticeably higher across all portfolios. The win rate remains relatively stable, with minor improvements for the Adaptive LASSO portfolios when transaction costs are excluded. The Sharpe Ratio (SR) and Sortino Ratio (SOR) improve without transaction costs. The Adaptive LASSO SR portfolio's SR rises from 76.74% (with costs) to 80.73% (without costs), and its Sortino Ratio increases from 255.34% to 267.55%. These improvements indicate that transaction costs erode a portion of the risk-adjusted returns.

Table 3 - Performance without transaction costs

	S&P 500	**1/N**	**Adaptive LASSO 1/N**	**Adaptive LASSO SR**
Win rate	68.24	68.24	67.45	65.10
CumRet	1.8974	2.1547	2.8849	3.5666
Skewness	-0.4753	-0.3399	0.5154**	0.5986***
Annualized mean return	11.13	12.67	16.70*	21.12**
Annualized Standard Deviation	16.44	17.73	20.02	24.62
SR	60.25	64.54	77.20	80.73
SOR	129.74	145.26	222.67	267.55**
CE ($\gamma = 1$)	9.67	10.98	14.71*	18.18**
CE ($\gamma = 3$)	6.70	7.59	11.49	13.56*
CE ($\gamma = 5$)	2.65	2.82	6.74	6.85
$CVaR_{1\%}$	21.48	22.83	18.05	21.38
$CVaR_{5\%}$	12.02	12.06	9.99**	11.62
MD	52.15	49.89	35.81	42.01

Notes: This table presents the performance metrics of the S&P 500 and 1/N benchmark portfolios and the two proposed trading strategies (Adaptative LASSO 1/N and Adaptative LASSO SR), without considering transaction costs. For a detailed description of the metrics presented see the notes of Table 2. All values are presented in percentage.

Figure 2 - Cumulative returns with different proportional transaction costs

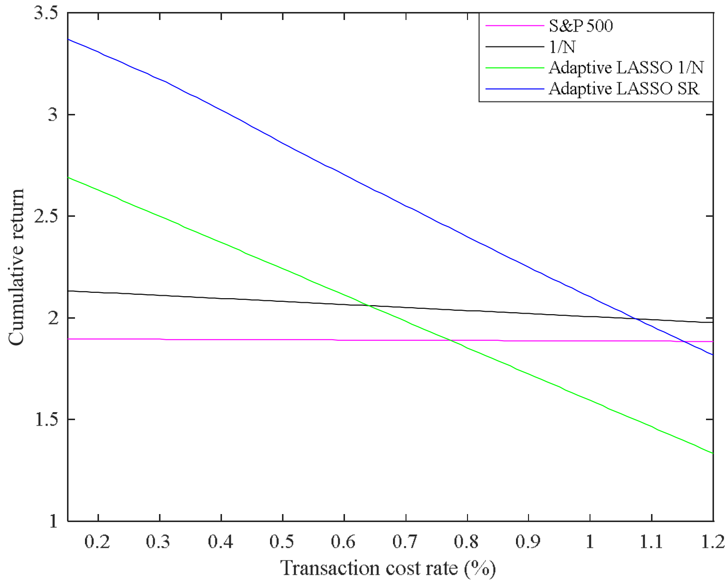

Figure 2 illustrates the cumulative returns of various portfolios with different levels of proportional transaction costs. To explore how transaction costs affect portfolio performance, the portfolios were constructed for transaction cost rates ranging from 0.15% to 1.2%, with intervals of 0.01%. The Adaptive LASSO 1/N portfolio is the most impacted, with its cumulative returns falling below the 1/N benchmark when transaction costs exceed 0.64%. On the other hand, the Adaptive LASSO SR portfolio continues to outperform the 1/N benchmark until the transaction cost reaches 1.07% (break-even transaction cost), i.e. a round-trip cost of 2.14%.

Overall, while transaction costs do reduce cumulative returns, the Adaptive LASSO portfolios, particularly the SR version, remain competitive even at higher cost levels.

5.4. Sensitivity enter/exit the market threshold

Table 4 presents the results of a sensitivity analysis to the entry/exit threshold. The results support the robustness of the Adaptive LASSO strategies.

Table 4 - Sensitivity analysis to market entry and exit thresholds

Threshold	Performance metrics	Adaptive LASSO 1/N	Adaptive LASSO SR
0%	CumRet	271.15	**340.52**
	Mean return	15.82	**20.27****
	SR	72.55	**77.63**
	SOR	220.01*	**259.94****
0.05%	CumRet	**273.44**	339.45
	Mean return	**15.94**	20.24**
	SR	**73.21**	77.04
	SOR	**222.21****	259.65**
0.1%	CumRet	268.05	339.50
	Mean return	15.65	20.23**
	SR	72.00	77.16
	SOR	216.53*	256.78**
0.15%	CumRet.	269.22	337.20
	Mean return	15.70	20.11**
	SR	72.41	76.74
	SOR	214.74*	255.34**
0.2%	CumRet.	264.83	334.88
	Mean return	15.48	19.99**
	SR	71.17	76.25
	SOR	218.56*	256.63**
0.25%	CumRet.	233.95	301.97
	Mean return	13.91	18.30**
	SR	63.25	69.40
	SOR	187.54*	229.84**
0.3%	CumRet	234.13	301.93
	Mean return	13.91	18.29**
	SR	63.33	69.44
	SOR	188.33*	229.87**

Notes: This table presents the annualized cumulative return, the annualized mean return, Sharpe ratio and Sortino ratio. All metrics are computed using round-trip transaction costs of 0.3%. The p-values are obtained using 100,000 bootstrap samples created with the circular block procedure of Politis and Romano (1994), with an optimal block size chosen according to Politis and White (2004) and Politis and White (2009). "*", "**" and "***" denote the significance levels of 10%, 5% and 1%, respectively. In bold are the best results for the two LASSO approaches. All values are presented in percentage.

The optimal threshold for the Adaptive LASSO SR portfolio is 0%, yielding a cumulative return of 340.52%, an annualized Sharpe ratio of 77.63%, and an annualized Sortino ratio of 259.94%. For the Adaptive LASSO 1/N portfolio, the best performance is achieved at a threshold of 0.05%, with a cumulative return of 273.44%, an annualized mean return of 15.94%, an annualized Sharpe ratio of 73.21%, and an annualized Sortino ratio of 222.21%. These low values for the optimal thresholds may result from the general upward trend in stock prices over the period considered, making it usually worthwhile to stay in the market despite transaction costs.

While the threshold choice influences portfolio performance, its impact is not highly pronounced. Both Adaptive LASSO portfolios maintain a statistically significant advantage over the 1/N benchmark in terms of the Sortino ratio across all thresholds. The Adaptive LASSO SR portfolio also consistently exhibits a statistically higher monthly mean return compared to the 1/N benchmark; a distinction not always observed for the Adaptive LASSO 1/N portfolio.

These findings highlight the resilience of the Adaptive LASSO SR strategy in generating superior returns under varying market conditions. The variation in entry/exit thresholds does not substantially alter the strategy's performance, indicating that strict threshold optimization may not be necessary. Instead, investors may prioritize other factors, such as transaction costs and liquidity constraints, when selecting an entry/exit threshold. The overall results underscore the potential of the Adaptive LASSO framework to deliver strong, risk-adjusted returns while maintaining robustness across different market conditions.

6. Conclusions

This study explores the impact of various firm-specific features on forecasting returns for a large set of US stocks traded on the NYSE and NASDAQ between January 1990 and December 2022. By applying the adaptive LASSO methodology, we identify the most relevant features for predicting stock returns and use these forecasts to construct two threshold-based portfolios: Adaptive LASSO 1/N, with equal weights for selected stocks, and Adaptive LASSO SR, where stock weights are determined by their expected Sharpe ratio. Additionally, we incorporate a round-trip transaction cost of 0.3% and a no-trade region to manage turnover rates and mitigate trading costs.

In terms of feature relevance, the adaptive LASSO consistently selects firm characteristics such as lagged returns, log-transformed trading volumes, market value, employee count, dividend yield, R&D expenditure relative to sales, net cash flow investing divided by market value, and return on invested capital. These features are selected more than 10% of the time across the sample period, with some reaching a selection frequency of up to 51%.

The portfolio analysis reveals that the Adaptive LASSO SR portfolio outperforms its counterparts, achieving a cumulative return of 337.20%, which substantially exceeds the S&P 500 (189.57%) and 1/N (213.24%) benchmarks. Moreover, the Adaptive LASSO SR portfolio has the highest annualized Sharpe ratio at 76.74%, surpassing the other portfolios, which further emphasizes the superior risk-adjusted performance of the adaptive LASSO-based strategies. These findings affirm that adaptive LASSO predictions contribute significantly to portfolio performance, providing a reliable method for selecting profitable stocks.

Furthermore, the sensitivity analysis shows that the break-even transaction cost for the Adaptive LASSO SR portfolio is estimated to be 1.07%. This indicates that the strategy remains robust even with moderate trading costs. The combination of threshold-based strategies with the no-trade boundary effectively reduces turnover and enhances the efficiency of portfolio construction.

Additionally, sensitivity to entry/exit thresholds demonstrates that the Adaptive LASSO SR portfolio consistently maintains superior performance across a range of thresholds, with the optimal threshold set at 0%. This threshold yields a cumulative return of 340.52% and an annualized Sharpe ratio of 77.63%, solidifying the Adaptive LASSO SR portfolio as the best-performing strategy under varying market conditions.

In conclusion, this study highlights the effectiveness of adaptive LASSO in identifying key firm characteristics for stock return forecasting and constructing portfolios that outperform traditional benchmarks, even when accounting for transaction costs.

Declarations

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Conflicts of interest/Competing interests

The authors have no relevant financial or non-financial interests to disclose.

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Table A.1 – List of the features

#	Acronym	Description
1	Amihud	Amihud ratio (see Equation (1))
2	Returns	Arithmetic returns, given by $r_{i,t} = \frac{TRI_{i,t}}{TRI_{i,t-1}} - 1$, where $r_{i,t}$ is the arithmetic return for stock i at time t and $TRI_{i,t}$ is the total return index for stock i at time t
3	Cf_per_sales	Cash flows-to-sales, given by the ratio between funds from operations and net sales or revenues, multiplied by 100
4	Current_ratio	Current ratio, given by the ratio between total current assets and total of current liabilities
5	Dep_div_mv	Depreciation and depletion divided by market value, where the market value is given by share price multiplied by number of ordinary shares in issue
6	Div_yield	Dividend yield, given by the ratio between dividend per share and share price, multiplied by 100
7	Emp_growth	Employees 1-year yearly growth, given by the current year's total employees divided by last year's total employees minus 1, all multiplied by 100
8	EPS_div_Price	Earnings per share divided by the adjusted closing price, where the earnings per share are given by the revenue multiplied by the net margin and divided by the number of shares outstanding
9	Gross_profit_margin	Gross profit margin, given by the ratio between gross income and net sales or revenues, multiplied by 100
10	Income_tax_div_mv	Income taxes divided by market value, where the income taxes correspond to all income taxes levied on the income of a company by federal, state, and foreign governments and the market value is given by share price multiplied by number of ordinary shares in issue
11	Inv_turnover	Inventory turnover, that corresponds to the ratio between the cost of goods sold (excluding depreciation) and average of inventories of the last and current years
12	Log_Employees	Natural logarithm of the number of both full and part-time employees of the firm
13	Log_mv	Natural logarithm of market value, where the market value is given by share price multiplied by number of ordinary shares in issue
14	Log_Vol	Natural logarithm of volumes, where volume correspond to the adjusted closing price, multiplied by turnover by volume, where the turnover by volume is the number of shares traded for a stock on a particular day
15	Mean_Amihud	Mean of current and three previous values of Amihud illiquidity ratio
16	Mean_log_Vol	Mean of current and three previous values of log-volumes
17	Mean_Parkinson	Mean of current and three previous values of Parkinson range volatility estimator
18	Net_cf_financing	Net cash flow – financing divided by market value, where the net cash flow – financing corresponds to net cash receipts and disbursements resulting from reduction and/or increase in long- or short-term debt, proceeds from the sale of stock, stock repurchased/redeemed/retired, dividends paid, and other financing activities

19	Net_cf_investing	Net cash flow – investing divided by market value, where the net cash flow – investing corresponds to net cash receipts and disbursements resulting from capital expenditures, decrease/increase from investments, disposal of fixed assets, increase in other assets and other investing activities
20	Net_debt_div_mv	Net debt divided by market value, where the net debt is the difference between total debt and cash and the market value is given by share price multiplied by the number of ordinary shares in issue
21	Net_margin_div_mv	Net margin divided by market value, where the net margin is the ratio between net income and net sales or revenues, multiplied by 100 and the market value is given by share price multiplied by the number of ordinary shares in issue
22	Net_sales_div_mv	Net sales or revenues divided by market value, where the net sales or revenues are the gross sales and other operating revenue, excluding discounts, returns and allowances and the market value is given by share price multiplied by the number of ordinary shares in issue
23	OPM_div_mv	Operating profit margin divided by market value, where the operating profit margin is the ratio between operating income and net sales or revenues, multiplied by 100, and the market value is given by share price multiplied by the number of ordinary shares in issue
24	Parkinson	Parkinson volatility range estimator (see Equation (2))
25	Price_to_book	Price-to-book value, given by share price divided by the book value per share
26	PER	Price-to-earnings ratio, given by adjusted closing price divided by the earnings rate per share
27	Quick_ratio	Quick ratio, given by cash and equivalents plus receivables (net) divided by total current liabilities
28	RD_to_sales	Research and development expenses-to-sales, given by research and development expense divided by net sales or revenues, multiplied by 100
29	Return_on_equity	Return on equity, given by ((net income - preferred dividend requirement) / average of last year and current year's common equity) * 100
30	Return_on_inv_cap	Return on invested capital, given by (net income + ((interest expense on debt - interest capitalized) * (1-tax rate))) / (average of last year and current year's (total capital + short term debt and current portion of long-term debt))*100
31	Sps_div_Price	Sales per share divided by the adjusted closing price, where sales per share correspond to the per share amount of the company's sales or revenues for the 12 months ended the last calendar quarter of the year
32	SGAE_to_sales	Selling, general and administrative expenses-to-sales, where the selling, general and administrative expenses correspond to ((selling, general and administrative expenses - research and development expense) / net sales or revenues)) * 100
33	Share_turn	Share turnover rate, given by the ratio between trading volume and market capitalization, multiplied by 100
34	Shareholders_equity_div_mv	Total shareholders' equity divided by market value, where the total shareholders' equity is the sum of preferred stock and common shareholders' equity, and the market value is given by share price multiplied by number of ordinary shares in issue
35	Total_asset_turn	Total asset turnover, given by the ratio between net sales or revenues and total assets

36	Total_debt_of _common_equity	Total debt (% of common equity), given by ((long-term debt + short-term debt and current portion of long-term debt) / common equity)*100
37	Total_debt_of_total_cap	Total debt (% of the total capital), given by long-term debt + short-term debt and current portion of long-term debt) / (total capital + short-term debt and current portion of long-term debt)*100
38	Total_liabilities_div_mv	Total liabilities divided by market value, where the total liabilities are all short- and long-term obligations expected to be satisfied by the company and the market value is given by share price multiplied by the number of ordinary shares in issue
39	Work_cap_div_mv	Working capital divided by the market value, that is, the difference between the current assets and current liabilities, divided by share price multiplied by number of ordinary shares in issue

Notes: This table presents the acronyms of the variables, in alphabetical order, used in the present paper, their summary description, mostly based on the notes available in Refinitiv Eikon database.

Table A.2 – Features selection by the adaptive LASSO

Variable	Relative number of months where each feature is selected by adaptive LASSO (%)		
	Overall sample	**In-sample**	**Out-of-sample**
Returns(1)	**29.18**	**35.29**	**25.10**
Returns(2)	**31.29**	**33.53**	**29.80**
Returns(3)	**29.18**	**28.82**	**29.41**
Returns(4)	**31.06**	**28.82**	**32.55**
Amihud	23.29	26.47	21.18
Cf_per_sales	16.24	20.59	13.33
Current_ratio	23.53	21.18	25.10
Dep_div_MV	27.29	26.47	27.84
Div_yield	**31.77**	**32.94**	**30.98**
Emp_growth	17.18	16.47	17.65
EPS_div_Price	22.35	20.59	23.53
Gross_profit_margin	28.24	25.29	**30.20**
Income_tax_div_MV	18.59	19.41	18.04
Inv_turnover	24.94	26.47	23.92
Log_Employees	**32.24**	**35.29**	**30.20**
Log_mv	**38.59**	**33.53**	**41.96**
Log_Vol	**39.77**	**31.18**	**45.49**
Mean_Amihud	23.29	24.12	22.75
Mean_Parkinson	27.53	**30.00**	25.88
Mean_log_Vol	**46.35**	**40.00**	**50.59**
Net_cf_financing	17.41	24.12	12.94
Net_cf_investing	**31.53**	**34.71**	**29.41**
Net_debt_div_mv	21.65	20.00	22.75
Net_margin_div_mv	26.12	24.12	27.45
Net_sales_div_mv	26.12	25.29	26.67
OPM_div_mv	25.88	23.53	27.45
Parkinson	19.29	16.47	21.18
Price_to_book	14.35	17.65	12.16
PER	15.53	10.59	18.82
Quick_ratio	**31.53**	**28.82**	**33.33**
RD_to_sales	**32.47**	**31.77**	**32.94**
Return_on_equity	18.12	20.00	16.86
Return_on_inv_cap	**29.41**	**35.88**	25.10
Sps_div_Price	9.88	14.71	6.67
SGAE_div_sales	20.24	19.41	20.78
Share_turn	14.59	**30.00**	4.31
Shareholders_equity_div_mv	20.24	18.24	21.57
Total_asset_turn	21.65	24.71	19.61
Total_debt_of_common_equity	20.71	16.47	23.53
Total_debt_of_total_cap	18.35	19.41	17.65
Total_liabilities_div_mv	25.41	25.29	25.49
Work_cap_div_mv	19.53	11.77	24.71

Notes: This table reports the relative number of months in the overall sample and in the in-sample and out-of-sample subperiods, in which the adaptive LASSO selects each feature for forecasting the returns. All the variables are lagged one month, except Returns(k) that are lagged k months. In bold are the features chosen in at least 25% of the months. The values are presented in percentages.